RECURSIONS FOR THE INDIVIDUAL MODEL

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Abstract: Recently, Waldmann considered an algorithm to compute the aggregate claims distribution in the individual life model which is an efficient reformulation of the original exact algorithm of De Pril.

In this paper we will show that in practice the approximations as proposed by De Pril are still more efficient than the exact algorithm of Waldmann both in terms of the number of computations required and of the memory occupied by intermediate results.

Furthermore we will generalize the algorithm of Waldmann to arbitrary claim amount distributions. We will compare this algorithm with respect to efficiency with the algorithms that were derived by De Pril for this model. It turns out that the approximations of De Pril are most efficient for practical computations.

Keywords: recursion formula, aggregate claims distribution, arbitrary claim amount, efficiency, individual model

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1 Introduction

Consider a portfolio of independent policies and suppose that the probability of producing a claim in a certain reference period, e.g. one year, and the associated claim amount distribution are given for each policy. During the past decade recursive computation of the aggregate claims distribution of such a portfolio has frequently been dealt with in the actuarial literature.

The case where the individual claim amounts can only take 2 values, 0 and some strictly positive value, has been called the *individual life model* because of its straightforward application in life insurance. This model is considered in De Pril (1986) where a recursion for exact computation of the aggregate claims distribution is derived. An efficient reformulation of this algorithm is given in Waldmann (1993).

Exact recursive procedures for computing the aggregate claims distribution in the individual model with arbitrary positive claim amounts –in the sequel referred to by *the individual model*– are derived in De Pril (1989).

In the present paper, a new recursive scheme is proposed for exact computation of the aggregate claims distribution in the individual model. Waldmann's recursion for the individual life model is found as a special case.

A comparison of the different exact procedures for the individual model reveals that the one derived in this paper is to be preferred if the portfolio is not too heterogeneous and if the number of arithmetical operations to be carried out and the data to be kept for further computations are used as measures of efficiency.

In De Pril (1988, 1989) and Dhaene and De Pril (1994) recursive procedures are considered for approximate computation of the aggregate claims distribution. In this paper, a comparison is made between these approximate procedures and the exact procedure for the individual model. It turns out that the r-th order approximations of De Pril will still perform best in most practical situations.

Kaas (1993) states that several kinds of error have to be considered when computing the aggregate claims distribution. A first type of error results from rounding the possible claim amounts of the policies to some monetary unit, e.g. \$ 1000. Computing the aggregate claims distribution of this –possibly rounded– portfolio generates a second type of error if this computation is done approximately (e.g. compound Poisson approximation, De Pril's r-th order approximation, ...).

Both types of error can be reduced at the cost of extra computing time. It is of course useless to apply an algorithm that computes the distribution function exactly if the monetary unit is large. On the other hand, higher order approximations of De Pril can be used if the monetary unit can be chosen sufficiently small.

Bounds for the different types of error are helpful in fixing the monetary unit and choosing between the algorithms for the rounded model. Bounds for the first type of error can be found in Goovaerts, et al. (1990). Bounds for the second type of error are considered in De Pril (1989) and Dhaene and De Pril (1994).

In this paper we will focus on the second type of error by starting from the rounded claim amounts. The term *exact algorithm* should be interpreted as an algorithm that doesn't produce the second type of error, whereas an *approximate algorithm* stands for an approximate computation of the aggregate claims distribution for the rounded model.

2 Model description

Consider a portfolio of n independent policies. This portfolio is divided into a number of classes by gathering all policies with the same claim probability and the same conditional claim amount distribution. The conditional claim amount distribution of a policy is the distribution of the claim amounts given that a claim occurs.

		claim probability					
		q_1	q_2	$\dots$	q_j	$\dots$	q_b
	$f_1(x)$						
	$f_2(x)$				$\vdots$		
claim	$\vdots$						
amount	$f_i(x)$			$\dots$	n_{ij}		
distribution	$\vdots$						
	$f_a(x)$						

More formally, the class (i, j), $i = 1 \dots a$, $j = 1 \dots b$ contains all policies with claim probability q_j and conditional claim distribution $f_i(x)$. It is assumed that for each j, $0 < q_j < 1$ and that the claim amounts of the individual policies are integral multiples of some convenient monetary unit so that for each i, $f_i(x)$ is defined for $x = 1, 2, \dots m_i$.

Further the following notation is used

- n_{ij}: number of policies in class (i, j)
- n_j: number of policies with the same claim probability
- $p_j = 1 - q_j$: probability that a policy in class (i, j) produces no claim
- $M = \sum_{i=1}^{a} \sum_{j=1}^{b} n_{ij}\ m_i$: maximal amount of aggregate claims

The probability that the total amount of claims produced by the portfolio during the exposure period equals s is denoted by $p(s)$. It follows immediately that $p(s)$ is the probability function of a random variable with range $0, 1, \dots M$.

The individual life model can be obtained from the individual model considered above by choosing $f_i(x) = \delta_{ix}$, $i = 1, 2, \dots a$ where $\delta_{ix} = 1$ if $i = x$ and 0 otherwise.

3 Recursions for the individual life model

In Waldmann (1993) the following exact recursion scheme for computing the aggregate claims distribution in the individual life model is proposed:

Theorem 1: (Waldmann)

For the individual life model the probabilities $p(s)$ can be computed by

$$p(0) = \prod_{i=1}^{a} \prod_{j=1}^{b} (p_j)^{n_{ij}} \tag{1}$$

$$s\ p(s) = \sum_{i=1}^{a} \sum_{j=1}^{b} i\ n_{ij}\ t_{ij}(s) \text{ for } s = 1, 2, \ldots M \tag{2}$$

where the coefficients $t_{ij}(s)$ are given by

$$t_{ij}(s) = \frac{q_j}{p_j} \left(p(s-i) - t_{ij}(s-i)\right) \text{ for } s = 1, 2, \ldots M \tag{3}$$

and $t_{ij}(s) = 0$ elsewhere.

Waldmann obtains his result by rearrangement of the recursive scheme of De Pril (1986) and shows that this exact recursive scheme is more efficient than the original algorithm of De Pril by comparing the number of arithmetic operations to be carried out and the number of data to be kept at each stage of the iteration for both methods.

In order to reduce computing time, De Pril (1988) developed formulae to approximate the exact probabilities $p(s)$ if the claim probabilities q_j are smaller than $\frac{1}{2}$. The r-th order approximations $p^{(r)}(s)$ suggested by him are computed recursively as follows:

Theorem 2: (De Pril)

For the individual life model the probabilities $p(s)$ can be computed approximately by

$$p^{(r)}(0) = p(0) \tag{4}$$

$$s\ p^{(r)}(s) = \sum_{i=1}^{min(a,s)} \sum_{k=1}^{min(r, \lfloor \frac{s}{i} \rfloor)} a(i,k)\ p(s-ki) \text{ for } s = 1, 2, \ldots M \tag{5}$$

where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x and the coefficients $a(i,k)$ are given by

$$a(i,k) = (-1)^{k+1} i \sum_{j=1}^{b} n_{ij} \left(\frac{q_j}{p_j}\right)^k. \tag{6}$$

This way, an infinite number of approximations is defined. From De Pril (1988) it follows that increasing the order r will lead to better approximations. If $r = M$ the recursion scheme of Theorem 2 equals the exact recursive scheme proposed in De Pril (1986).

In Waldmann (1993) a comparison is made between the performance of the algorithm of Theorem 1 and the algorithm of Theorem 2 for $r = M$. From theoretical error bounds for the r-th order approximations, see De Pril (1989) and Dhaene and De Pril (1994), but also from numerical examples, see Vandebroek and De Pril (1988), it follows that in practical applications, a value of r equal to $3, 4$ or 5 will give almost exact results. A question that naturally arises then is how the De Pril approximations, with r small, perform in comparison with Waldmann's exact recursion.

In order to answer this question, we will first focus on the number of arithmetical operations to be carried out for computing the probabilities that the aggregate claims equal $1, 2, \ldots, s$ with s large.

Waldmann (1993) considered only the number of arithmetical operations to be carried out for one step of the iteration, i.e. for computing a single $p(s)$-value. In practice the insurer will not be interested in a single probability but in the whole distribution function of the aggregate claims. It is however not necessary to compute the probablities $p(s)$ for very large s-values, i.e. close to M, because these are too small to be distinguished from zero.

An algorithm to compute the distribution function up to a value s is said to be $O(s^m)$ if the number of multiplications required depends on $s, s^2, s^3, \ldots, s^m$ and on no higher order terms in s. We will determine the order of the algorithms and compare the algorithms of equal order in more detail. An advantage of tackling the problem this way is that the initial work needed to start the algorithm can be neglected as it is independent of s.

For the algorithm in Theorem 1 the number of multiplications required to compute $p(1), p(2), \ldots, p(s)$ is approximately equal to $2\ a\ b\ s$. Remark that in counting the number of multiplications we have assumed that the quantities $i\ n_{ij}$ and $\frac{q_j}{p_j}$ are stored. Similarly, De Pril's r-th order method, with r small, requires approximately $2\ a\ r\ s$ multiplications for computing $p^{(r)}(1), p^{(r)}(2), \ldots, p^{(r)}(s)$.

Remark that we have not counted the number of multiplications needed to compute the coefficients $a(i,k)$, $i = 1\ldots,a$ and $k = 1\ldots r$, in De Pril's method because this number is independent of s and will therefore be negligible in our comparison.

Comparing these values reveals that the number of multiplications is lower for the r-th order approximation method if

$$r < b. \tag{7}$$

In practical situations, this inequality will be satisfied because an r-value of 3 or 5 will be sufficient to achieve almost exact results and the number of different claim probabilities b will typically be 20 or more.

From the reasoning above, it follows immediately that Waldmann's exact recursion and De Pril's r-th order method are $O(s)$. As the approximating algorithm gives exact results for $p^{(r)}(1),\ldots,p^{(r)}(s)$ if $r \geq s$ it is easy to prove that De Pril's exact algorithm is $O(s^2)$.

The number of additions to be carried out for both methods leads to similar results.

Further, we will also compare the number of data to be kept for further computation. Using Theorem 1 an array with $a(a+1)\frac{b}{2}$ elements is required, see Waldmann (1993). Using Theorem 2 to compute the approximations $p^{(r)}(1), p^{(r)}(2),\ldots,p^{(r)}(s)$ all the $a(i,k)$, $i = 1\ldots,a$, $k = 1\ldots r$ are needed. This means that $a\ r$ numbers have to be stored. It follows that the r-th order approximation will occupy less memory if

$$r < \frac{b}{2}(a+1). \tag{8}$$

In practical situations, this inequality will again be satisfied.

We can conclude that although the exact iterative scheme for the individual life model, as proposed by Waldmann (1993) is much more efficient than the one proposed by De Pril (1986), the r-th order approximations of De Pril (1989) seem to perform better in many practical situations.

4 Recursions for the individual model with arbitrary claim amounts

4.1 A new recursive scheme

An exact recursion formula for computing the aggregate claims distribution in the individual model with arbitrary positive claims is given in the following theorem

Theorem 3:

For the individual model the probabilities $p(s)$ can be computed by

$$p(0) = \prod_{i=1}^{a}\prod_{j=1}^{b}(p_j)^{n_{ij}} \tag{9}$$

$$s\ p(s) = \sum_{i=1}^{a}\sum_{j=1}^{b} n_{ij}\ v_{ij}(s) \text{ for } \ s = 1, 2, \ldots M \tag{10}$$

where the coefficients $v_{ij}(s)$ are given by

$$v_{ij}(s) = \frac{q_j}{p_j}\sum_{x=1}^{m_i} f_i(x)\,(\ x\ p(s-x) - v_{ij}(s-x)) \text{ for } \ s = 1, 2, \ldots M \tag{11}$$

and $v_{ij}(s) = 0$ elsewhere.

Proof: The probability generating function of the aggregate claims is given by

$$P(u) = \sum_{s=0}^{M} p(s)\ u^s \tag{12}$$

$$= \prod_{i=1}^{a}\prod_{j=1}^{b}(p_j + q_j G_i(u))^{n_{ij}} \tag{13}$$

with $G_i(u)$ the generating function of the $f_i(x)$

$$G_i(u) = \sum_{x=1}^{m_i} f_i(x)\ u^x. \tag{14}$$

Putting $u = 0$ in (14) leads to (9). In order to prove (10), take the derivative of (13)

$$P'(u) = \sum_{i=1}^{a} \sum_{j=1}^{b} n_{ij} V_{ij}(u) \tag{15}$$

where the auxiliary functions $V_{ij}(u)$ are defined by

$$\begin{aligned} V_{ij}(u) &= \sum_{s=0}^{\infty} v_{ij}(s+1) u^s & (16) \\ &= \frac{q_j \; G_i'(u) \; P(u)}{p_j + q_j \; G_i(u)}. & (17) \end{aligned}$$

Taking the derivative of order $(s-1)$ and inserting $u = 0$ yields (10). The recursion formula for the $v_{ij}(s)$ is obtained by differentiating the following expression $(s-1)$ times

$$V_{ij}(u) = \frac{q_j}{p_j} \; (G_i'(u) \; P(u) - G_i(u) \; V_{ij}(u)) \tag{18}$$

and putting $u = 0$. This completes the proof. ∎

The iterative procedure of Waldmann (1993) for the individual life model can be obtained from Theorem 3 with $f_i(x) = \delta_{ix}$. It then follows that the probabilities $p(s)$ can be computed by (10) with the $v_{ij}(s)$ given by

$$v_{ij}(s) = \frac{q_j}{p_j} \, (\; i \; p(s-i) - v_{ij}(s-i)) \;\; \text{for } s = 1, 2, \ldots M. \tag{19}$$

Now let $t_{ij}(s)$ be defined by

$$t_{ij}(s) = v_{ij}(s)/i \tag{20}$$

then (2) is obtained.

So, Theorem 3 is indeed a generalization of Waldmann's recursive procedure for the individual life model. Waldmann proved his result by rearranging the coefficients of De Pril's exact algorithm for the individual life model. For the general case of Theorem 3, it seems impossible to obtain the recursion (10) by simple rearrangement of one of the exact algorithms for the individual model considered in De Pril (1989).

Remark: Several authors propose to approximate the distribution of the aggregate claims by a compound Poisson distribution with parameter

$$\lambda = \sum_{i=1}^{a} \sum_{j=1}^{b} n_{ij} \; q_j \tag{21}$$

and claim amount distribution $f(x)$ given by

$$f(x) = \frac{\sum_{i=1}^{a} \sum_{j=1}^{b} n_{ij} \; q_j \; f_i(x)}{\lambda} \tag{22}$$

see e.g. Gerber (1984) or De Pril and Dhaene (1994).

Using Panjer's recursion formula one finds that the approximated probabilities $p^{cp}(s)$ can here be computed by

$$p^{cp}(0) = e^{-\lambda} \tag{23}$$

$$s \; p^{cp}(s) = \sum_{i=1}^{a} \sum_{j=1}^{b} n_{ij} \; v_{ij}(s) \quad \text{for } s = 1, 2, \ldots \tag{24}$$

where the coefficients $v_{ij}(s)$ are defined by

$$v_{ij}(s) = q_j \sum_{x=1}^{m_i} f_i(x) \; x \; p^{cp}(s-x) \quad \text{for } s = 1, 2 \ldots \tag{25}$$

Remark the similarity between this approximate procedure and the exact procedure given in Theorem 3.

Example 1: Consider a life coverage with double indemnity provision providing a death benefit which is doubled when death is caused by an accident. The conditional claim amounts distribution is then given by

$$f_i(x) = \begin{cases} 1-\alpha & x = i \\ \alpha & x = 2i \\ 0 & \text{elsewhere} \end{cases} \tag{26}$$

so that the aggregate claims distribution can be computed by Theorem 3 with

$$v_{ij}(s) = \frac{q_j}{p_j} \ \{(1-\alpha)\,[i\ p(s-i) - v_{ij}(s-i)] + \alpha\,[2\ i\ p(s-2i) - v_{ij}(s-2i)]\} \tag{27}$$

This is a much simpler algorithm than the analogous one in De Pril (1989).

Example 2: Consider a portfolio consisting of n policies with identical claim probability distributions. In this case there is only one class and as such the indices i and j can be omitted. It follows from Theorem 3 that

$$p(0) = p^n \tag{28}$$

$$s\ p(s) = n\ v(s) \tag{29}$$

with

$$v(s) = \frac{q}{p} \sum_{x=1}^{m} f(x)\ (x\ p(s-x) - v(s-x)) \tag{30}$$

$$= \frac{q}{p} \sum_{x=1}^{m} f(x)\ \frac{(n+1)x - s}{n}\ p(s-x) \quad \text{for} \quad s = 1, 2, \ldots \tag{31}$$

which yields

$$s\ p(s) = \frac{q}{p} \sum_{x=1}^{m} f(x)\ \ ((n+1)x - s)\ \ p(s-x) \quad \text{for} \quad s = 1, 2, \ldots \tag{32}$$

This recursion, which is a special case of the recursion of Panjer (1981) for the compound binomial distribution can also be found in De Pril (1985).

4.2 Comparison with other recursive schemes

In this paragraph we will compare the performance of the new exact algorithm given in Theorem 3 with the performance of existing – exact and approximating– algorithms.

In De Pril (1989) the following recursion for computing the probabilities $p(s)$ in the individual model is proposed.

Theorem 4: (De Pril)

For the individual model the probabilities $p(s)$ can be computed by

$$p(0) = \prod_{i=1}^{a} \prod_{j=1}^{b} (p_j)^{n_{ij}} \tag{33}$$

$$sp(s) = \sum_{x=1}^{s} w(x)\ p(s-x) \text{ for } s = 1, 2, \ldots M \tag{34}$$

where the coefficients $w(x)$ are given by

$$w(x) = \sum_{i=1}^{a} \sum_{j=1}^{b}\ n_{ij}\ w_{ij}(x) \tag{35}$$

with

$$w_{ij}(x) = \frac{q_j}{p_j}\ \left(x\ f_i(x) - \sum_{k=1}^{m_i} f_i(k)\ w_{ij}(x-k) \right) \text{ for } \ x = 1, 2, \ldots M \tag{36}$$

and $w_{ij}(x) = 0$ elsewhere.

A nice property of this algorithm is the following: once the distribution function is computed for the portfolio under consideration, it is easy to compute the distribution function of a portfolio which consists of the same classes but where the number of policies n_{ij} are different.

In order to compare the performance of both exact procedures, at first the number of multiplications to be carried out for computing $p(1), p(2), \ldots p(s)$ with s large will be estimated for both methods.

The new procedure in Theorem 3 approximately leads to $2\ b\ (a + \ \sum_{i=1}^{a} m_i)$ multiplications for computing one probability $p(s)$. So computing $p(1), p(2), \ldots p(s)$ takes approximately $2\ b\ (a + \sum_{i=1}^{a} m_i)\ s$ multiplications.

On the other hand, the procedure in Theorem 4 leads to approximately $3\ a\ b + b\ \sum_{i=1}^{a} m_i + s$ multiplications for computing one probability $p(s)$. Computing the probabilities $p(1), \ldots, p(s)$ needs then $s\ \frac{(s+1)}{2} + b\ (3\ a + \sum_{i=1}^{a} m_i)s$ multiplications.

Comparing both numbers reveals that the algorithm in Theorem 3 performs better than the algorithm in Theorem 4 if

$$s > 2\ b\ (\sum_{i=1}^{a} m_i - a) - 1 \quad \left(> 2\ b\ \sum_{i=1}^{a} m_i \right). \tag{37}$$

In order to interpret this result we have to consider the degree of heterogeneity of the classified portfolio. We can get an idea of this degree by comparing $b\ \sum_{i=1}^{a} m_i$ with M. If the portfolio is extremely homogeneous there is only one class and $b\ \sum_{i=1}^{a} m_i << M$. If the portfolio is extremely heterogeneous the number of classes will be equal to the number of policies and $b\ \sum_{i=1}^{a} m_i = M$.

In practice, the degree of homogeneity of the portfolio will be somewhere between these two extremes.

From (37) we can conclude that Theorem 3 is an efficient reformulation of De Pril's Theorem 4 as to the number of multiplications to be carried out if the portfolio is not too heterogeneous. In this case the new algorithm is $O(s)$ whereas De Pril's algorithm is $O(s^2)$.

For very heterogeneous portfolios De Pril's method will have to be preferred.

Now the number of data-items to be kept for further iterations will be estimated for both procedures. Assume that the procedure of Theorem 3 is at stage s, this means that $p(1), p(2), \ldots,$ $p(s-1)$ have been computed. In order to compute $p(s)$ the coefficients $v_{ij}(s-x)$ with $i = 1, 2, \ldots a,\ j = 1, \ldots b$ and $x = 1, \ldots m_i$ have to be kept. This means that $b \sum_{i=1}^{a} m_i$ numbers have to be stored for each step of the iteration.

To apply the procedure in Theorem 4 $s\ (1 + a\ b)$ numbers have to be kept, i.e. all the coefficients $w_{ij}(x)$, $i = 1, 2, \ldots a$, $j = 1, \ldots b$, $x = 1, \ldots s$ and $w(x)$ for $x = 1, 2, \ldots s$. Comparing both figures reveals that the new procedure will always perform better with respect to this criterion.

We can conclude that in many cases the exact algorithm of Theorem 3 is more efficient than the exact algorithm of De Pril given in Theorem 4, considerably reducing both the number of arithmetical operations to be carried out and the number of data to be kept at each stage

of the iteration.

In order to reduce the considerable computing time that is required for the exact computations, De Pril (1989) proposed the following approximating algorithm which performs well if the q_j are smaller than $\frac{1}{2}$.

Theorem 5: (De Pril)

For the individual model the probabilities $p(s)$ can be computed approximately by

$$p^{(r)}(0) = \prod_{i=1}^{a}\prod_{j=1}^{b}(p_j)^{n_{ij}} \tag{38}$$

$$sp^{(r)}(s) = \sum_{i=1}^{a}\sum_{k=1}^{min(r,s)} A(i,k) \sum_{x=k}^{min(s,km_i)} xf_i^{*k}(x)\ p^{(r)}(s-x) \text{ for } s = 1,2,\ldots M \tag{39}$$

where the coefficients $A(i,k)$ are given by

$$A(i,k) = \frac{(-1)^{k+1}}{k} \sum_{j=1}^{b} n_{ij}\left(\frac{q_j}{p_j}\right)^k \tag{40}$$

and $f_i^{*k}(x)$ denotes the k-fold convolution of $f_i(x)$.

Remark that the first order approximation can also be obtained from Theorem 3 by approximating the $v_{ij}(s)$ defined in (11) by

$$v_{ij}^{(1)}(s) = \frac{q_j}{p_j}\sum_{x=1}^{m_i} xf_i(x)\ p(s-x) \tag{41}$$

Remark further that the algorithm in Theorem 5 leads to the exact probabilities $p(s)$ if $r \geq M$, see De Pril (1989). We will compare the performance of this algorithm, with r small, with the performance of the new algorithm of Theorem 3.

In order to compute $p^{(r)}(1), p^{(r)}(2), \ldots p^{(r)}(s)$ there are approximately

$$\left\{3\ a\ r + r\ (r+1)\sum_{i=1}^{a}(m_i - 1)\right\}\ s$$

multiplications required. In addition to this value there is some initial work needed to compute the coefficients $A(i,k)$ and the convolutions $f_i^{*k}(x)$ for $i = 1,\ldots a$, $k = 1,\ldots r$ and $x = 1,\ldots km_i$ but these calculations depend on r and not on s and can therefore be neglected here.

Comparison of the number of multiplications for the recursions of Theorem 3 and Theorem 5

reveals that both algorithms are $O(s)$ if $b\ \sum_{i=1}^{a} m_i << s$. Theorem 5 is preferred to Theorem 3 (w.r.t. the number of multiplications) if

$$\left\{3\ a\ r + r\ (r+1)\sum_{i=1}^{a}(m_i - 1)\right\}\ s < \left\{2\ b\ (a + \sum_{i=1}^{a} m_i)\right\}\ s. \tag{42}$$

This inequality can be transformed into

$$b > \frac{r\ (r+1)}{2} - \frac{a\ r}{2\ (\sum_{i=1}^{a} m_i + a)}(2\ r - 1) \quad \left(> \frac{r\ (r+1)}{2}\right) \tag{43}$$

which will be satisfied in practice.

Using Theorem 5 requires to keep all the $A(i,k)$ and the $f_i^{*k}(x)$ for $i = 1, \ldots a$, $k = 1 \ldots r$ and $x = 1 \ldots k\ m_i$. This means that an array of size $ar + \frac{r(r+1)}{2}\sum_{i=1}^{a}(m_i - 1)$ is required to keep these data.

Comparison of this number with the one obtained for the recursion in Theorem 3, reveals that the r-th order method of De Pril is preferred if

$$a\ r + \frac{r\ (r+1)}{2}\sum_{i=1}^{a}(m_i - 1) <\ b\ \sum_{i=1}^{a} m_i \tag{44}$$

This inequality can be transformed into

$$b > \frac{r\ (r+1)}{2} - \frac{a\ r}{2\ \sum_{i=1}^{a} m_i}(r - 1) \quad \left(> \frac{r\ (r+1)}{2}\right) \tag{45}$$

which is satisfied in most practical situations.

5 Conclusion

We showed that the original exact De Pril-algorithm for the individual model is $O(s^2)$ whereas his r-th order approximations are $O(s)$ if the portfolio is not too heterogeneous so that these methods reduce the required computation time drastically in this case. We derived a new exact algorithm for the individual model which is also $O(s)$ if the portfolio is not too

heterogeneous. Furthermore, this algorithm contains the Waldmann (1993) recursion for the individual life model as a special case.

Criteria were derived for choosing the optimal $O(s)$-algorithm, where optimality is measured by counting the number of multiplications required for computing the distribution function and by looking at the size of the array of the data to be kept.

The choice of the algorithm used in practical situations depends of course on the accuracy required, the computer capacity available, the size of the portfolio, ... and is as such very dependent on the situation considered. Nevertheless, the optimality criteria indicate that in a lot of practical situations, an r-th order approximation (with r small) will have to be preferred.

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