On the Characterization of Wang's Class of Premium Principles

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Abstract

A premium principle is an economic decison rule used by the insurer in order to determine the amount of the net premium for each risk in his portfolio. In this paper we investigate the problem of determining the premium principle to be used. First, we discuss some desirable properties of a premium principle. We prove that the only premium principles that possess these properties belong to a class of premium principles introduced by Wang (1996). Similar results can be found in Wang, Young & Panjer (1997).

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Introduction

From the point of view of the policyholder, an insurance system is a "mechanism for reducing the adverse financial impact of random events that prevent the fulfilment of reasonable expectations", see Bowers et al. (1986). This means that an insurance contract can be seen as a risk-exchange between two parties, the insurer and the policyholder. The insurer promises to pay for the financial consequences of the claims produced by the insured risk. In return for this coverage, the policyholder pays a fixed amount, called the premium. Observe that the payments made by the insurer are random, while the payments made by the policyholder are non-random.

The pure premium of the insured risk is defined as the expected value of the claim amounts to be paid by the insurer. In practice the insurer will add a risk loading to this pure premium. The sum of the pure premium and the risk loading is called the net premium. Adding aquisition and administration costs to this net premium, one gets the gross premium.

In this paper we will investigate the problem of determining the net premium. We will assume that the insurer adopts some economic decision rule to determine the amount of the net premium for each risk in his portfolio. Such a principle is called a premium principle, this is a rule that assigns a non-negative real number, the net premium, to each insured risk. Several premium principles have been presented in the actuarial literature. Wang (1996) remarks: "In insurance practice, the most widely used method is to base calculation on the first two moments. Since loss distributions are often highly skewed, the first two moments cannot rightly reflect the level of insurance risk." In the literature however, also other premium principles have been presented. Most of these methods have interpretations within the framework of expected utility theory. An overview is given in Goovaerts, De Vylder & Haezendonck (1984), see also Goovaerts, Kaas, van Heerwaarden & Bauwelinckx (1990).

Wang (1996) introduced a new class of premium principles which, loosely speaking, compute the net premium as the expectation of the risk under an adjusted measure. Wang's class of premium principles has close connections with Yaari's (1987) dual theory of choice under risk. It is also related to recent developments in non-additive measure theory, see Denneberg (1994). Actuarial applications of Wang's premium principle can be found in Wang & Dhaene (1997), and Dhaene, Wang, Young & Goovaerts (1997).

Recently, Wang, Young & Panjer (1997) take an axiomatic approach to characterize insurance prices in a competitive market setting. They determine some properties that should hold for a reasonable premium principle and prove that if these properties have to hold, then the premium principle that the insurer should use is uniquely determined. It turns out to be a principle belonging to Wang's class of premium principles. The main result of Wang, Young & Panjer (1997) is based on a representation theorem proved by Greco, see Denneberg (1994).

In this paper we start from a slightly different set of desirable properties for a premium principle. We give a simpler proof for the characterization of Wang's class of premium principles. On the other hand, we assume that the set of risks for which the premium principle is to be determined is broader than the one considered in Wang, Young & Panjer (1997). From a practical point of view, this seems not to be a restriction on the usability of our results, as in real life situations the class of risks for which the premium principle is to be determined will even be larger in most cases.

2 Properties of Premium Principles

A risk is defined as a non-negative real-valued random variables with finite mean, defined on some probability space. For each risk X we will denote its tail (or survival) function by S_X, i.e. $S_X(x) = \Pr[X > x]$, for all $x \geq 0$.

A special type of risks which will frequently be used in the sequel of this paper are the Bernouilli risks. For any $q \in [0,1]$, the probability function of the Bernouilli random variable B_q is given by $Pr[B_q = 1] = q = 1 - Pr[B_q = 1]$.

A lot of the existing actuarial models are built on the assumption that all risks in an insurance portfolio are mutually independent. In many real life insurance portfolios however, there will exist dependencies between certain risks, see eg. Dhaene & Goovaerts (1996, 1997). An extremal form of such a dependency relation is the comonotonicity of risks. Loosely speaking, two risks are comonotonic if they are bets on the same event, if neither of them is a hedge against the other, if both move in the same direction.

Definition 1 Two risks X and Y are comonotonic if, and only if, there exists a risk Z and non-decreasing functions f and g such that

$$(X,Y) \stackrel{\mathcal{D}}{=} (f(Z), g(Z)).$$

In the definition above, we used the notation $\stackrel{\mathcal{D}}{=}$ to indicate that the two bivariate random variables involved are equal in distribution. The concept of comonotonicty was introduced in the economic literature by Yaari (1987), see also Roëll (1987). Only recently, Wang (1996) introduced the concept of comonotonicity in the actuarial literature. Actuarial applications have been investigated in Wang & Dhaene (1997) and Dhaene, Wang, Young & Goovaerts (1997). Most insurance risk sharing schemes (between insurer and reinsurer, or between insurer and insured) lead to partial risks that are comonotonic. The only restriction that has to hold is that both risk sharing partners have to bear more if the underlying total claims increase.

Let Γ denote an appropriate set of risks for which the premiums have to be determined. Γ is assumed to be an "appropriate" set of risks which means that Γ has to be broad enough to contain all risks we need in the derivation of our results. In our case, this means that if $X \in \Gamma$, and d is an appropriate non-negative real number, then also $min(X, d)$ and dX are elements of Γ. Further, if $X \in \Gamma$ has bounded support on $[0, b]$, then also the random variables X_n $(n = 0, 1, 2, ...)$ defined by

$$X_n = \begin{cases} \frac{i}{2^n}b & : \frac{i}{2^n}b < X \leq \frac{i+1}{2^n}b;\ i = 0, 1, ..., 2^n - 1 \\ 0 & : elsewhere \end{cases}$$

are elements of Γ. Finally, for any $q \in [0, 1]$, the Bernouilli risk B_q defined above is an element of Γ.

Definition 2 A *premium principle* is a functional $H : \Gamma \rightarrow [0, \infty]$ that assigns to any risk $X \in \Gamma$, a non-negative real number, called the (net) premium $H(X)$. The premium is assumed to be equal for risks with the same distribution function.

Observe that premium can be infinite for some risks. These risks are called uninsurable for the principle under consideration.

In the remainder of this paper, we investigate the problem of determining a suited premium principle for Γ. We will solve this problem by considering a number of desirable properties for the premiums.

Each premium principle induces a total order between risks, ranking risks with a low premium below risks with a higher premium. A first desirable property of a premium principle is that the order obtained this way should correspond to one or more of the well-known stochastic orders between risks.

We say that a risk X is stochastically dominated by a risk Y if the tail probabilities are always higher for Y. We will consider premium principles for which stochastic dominance implies an ordering of the premiums.

Property 1 For any two risks X and Y in Γ we should have that $S_X(x) \leq S_Y(x)$ for all $x \geq 0$ implies $H(X) \leq H(Y)$.

The first property that we want to hold for a premium principle states that a risk that can be considered to be more "risky" than another one, should lead to a higher premium.

Consider the risks X, Y and Z in Γ. Assume that $X \stackrel{\mathcal{D}}{=} f(Z)$ and $Y \stackrel{\mathcal{D}}{=} g(Z)$ for non-decreasing functions f and g with $f+g$ equal to the identical function. Insuring X and Y separately leads to a premium income of $H(f(Z)) + H(g(Z))$, while insuring Z leads to a premium income of $H(f(Z) + g(Z))$. We assume that any rational insurer will prefer to insure X and Y separately instead of insuring Z because the risk Z can be considered as the sum of two comonotonic risks $f(Z)$ and $g(Z)$. Neither of these risks is a hedge against the other. No pooling effect is possible in this case. The insurer can incorporate this preference in the premium structure by requiring that $H(f(Z)) + H(g(Z)) \leq H(f(Z) + g(Z))$.

Knowing that the insurer charges a higher premium for insuring the complete risk Z than for insuring the parts $f(Z)$ and $g(Z)$ separately, a rational person will split his risk Z into $f(Z)$ and $g(Z)$ and buy separate policies for both comonotonic parts, assumed this strategy is possible. The insurer can avoid this situation by requiring that $H(f(Z) + g(Z)) \leq H(f(Z)) + H(g(Z))$.

We can conclude that if the insurer is not willing to give a reduction for a combined policy of comonotonic risks and if he wants to avoid splitting of risks, then he should use a premium principle such that $H(f(Z) + g(Z)) = H(f(Z)) + H(g(Z))$, which means that the premium principle should be additive for comonotonic risks. In the reasoning above, we implicitely assumed that it is possible to split risks. We restrict ourselves to types of risks for which this is possible.

Property 2 If risks X and Y in Γ are comonotonic, and if $X + Y \in \Gamma$, then we should have that

$$H(X + Y) = H(X) + H(Y).$$

Property 2 will not be a desirable property for all kinds of risks. Consider e.g. pricing the risk associated with a nuclear power installation. Assume that the whole risk X can be divided into pairwise comonotonic risks $X_1, ..., X_n$ with $X = X_1 + ... + X_n$. In such a situation, it is probable that a particular insurer is willing to insure a single part X_i at a premium $H(X_i)$, but he is only willing to insure a pair $X_i + X_j$ at a higher premium than the sum of the two individual premiums $H(X_i) + H(X_j)$, in order to be able to incorporate a higher risk load.

A thirth requirement that we impose on our premium principle arises from the fact that no risk loading is justified for a risk with variance equal to zero.

Property 3 Let 1 represent the degenerate risk which equals 1 with probability 1. Then we should have that $H(1) = 1$.

We will assume that the net premium is not lower than the pure premium. This means that the risk loading should be non-negative. This is a reasonable requirement for a premium principle. Indeed, consider a portfolio of n independent and identical distributed risks. Further assume that $H(X) < E(X)$ for all risks X. Then it can be shown that the probability that the premiums do not suffice to pay the aggregate claims goes to one if the size of the portfolio goes to infinity, see e.g. Sundt (1993).

Property 4 For any risk X in Γ we should have that $H(X) \geq E(X)$.

Let X be a risk in Γ, then also $\min(X, d)$ is a risk in Γ, for all $d \geq 0$. We have that $\min(X, d)$ converges in distribution to X if d goes to infinity. Hence, it seems reasonable to assume that also the premium of $\min(X, d)$ converges to the premium of X if d goes to infinity.

Property 5 For any risk X in Γ and $d \geq 0$ the functional H satisfies

$$\lim_{d \to \infty} H\left[\min(X, d)\right] = H(X).$$

The property above implies that the premium of X can be computed by approximating X by random variables with bounded support and taking the limit of the premiums for these random variables.

It is easy to prove that for a premium principle that satisfies Properties 1-3 we have that

$$H(aX + b) = aH(X) + b \text{ for all } a \geq 0 \text{ and } b \geq 0,$$

which means that such a premium principle is scale- and translation invariant.

3 Layers

Assume that a person originally bears a risk X. In practice it often happens that not the whole risk is insured, but only part of it. An example of such a coverage is a layer. The insurer will not have to pay a claim amount if X is less than or equal to some fixed amount a, called the deductible. Further a maximal intervention of the insurer, notation $b - a$, is stipulated in the policy. Finally, if X takes value in (a, b) then the payment of the insurer equals $X - a$. In this case the payment of the insurer is a random variable, notation $L(a, b)$, which is called the layer of X defined on (a, b).

Definition 3 Let $0 \leq a < b$. A layer at (a, b) of a risk X is defined as the loss from an excess-of-loss cover:

$$L(a,b) = \begin{cases} 0 & : 0 \leq X \leq a \\ X - a & : a < X < b \\ b - a & : X \geq b. \end{cases}$$

Layers are used because of several reasons. They often have a rather strong premium reduction effect. Further, as the policyholder has to bear part of the risk, this will force him to undertake prevention activities. In a reinsurance context a layer is called a stop-loss premium with retention a and maximal intervention $b - a$.

It is easy to verify that the tail function for the layer $L(a, b)$ is given by

$$S_{L(a,b)}(x) = \begin{cases} S_X(a + x) & : 0 \leq x < b - a \\ 0 & : x \geq b - a. \end{cases}$$

Now assume that X has bounded support $[0, b]$. Then it follows immediately that for any sequence $0 = x_0 < x_1 < x_2 < ... < x_n = b$ we have that X can be written as the following sum of layers:

$$X = \sum_{i=0}^{n-1} L(x_i, x_{i+1}).$$

Remark that the layers $L(x_i, x_{i+1})$, $i = 0, 1, ..., n-1$, are pairwise mutually comontonic risks.

Next, we define the notion of distortion function which arises in Yaari's (1987) dual theory of choice under risk and which can be seen as the parallel of the notion of utility function in expected utilty theory.

Definition 4 A function $g : [0, 1] \to [0, 1]$ is called a distortion function if g is non-decreasing with $g(0) = 0$ and $g(1) = 1$.

In the following lemma it is shown that a distortion function can be derived in a natural way from each premium principle satisfying the above mentionned properties.

Lemma 1 *If a premium principle $H : \Gamma \to [0, \infty]$ satisfies Properties 1-4, then the function g defined by*

$$g(q) = H(B_q), \qquad 0 \leq q \leq 1,$$

is a distortion function. Further, we have that $g(q) \geq q$ for all $q \in [0, 1]$.

Proof. It follows immediately that $g(0) = 0$ and $g(1) = 1$.

For $0 \leq q \leq p \leq 1$, we have that $S_{B_q}(x) \leq S_{B_p}(x)$ for all $x \geq 0$. Hence, we find that $g(q) \leq g(p)$, which means that g is non-decreasing.

For any $q \in [0, 1]$, we finally find that $g(q) = H(B_q) \geq E(B_q) = q$. ■

In the following section, we will prove that each premium principle satisfying Properties 1-5 is uniquely determined by the distortion function defined in Lemma 6.

4 Characterization of Wang's Class of Premium Principles

Wang (1996) proposes to compute the premium $H(X)$ of a risk X as follows:

$$H(X) = \int_0^\infty g\left[S_X(x)\right] dx,$$

where g is a distortion function with $g(q) \geq q$ for all $q \in [0, 1]$. As we have that $E(X) = \int_0^\infty \left[S_X(x)\right] dx$, we see that Wang proposes to compute the premium of X as a "distorted" expectation of X.

In this section, we prove a characterization theorem for Wang's class of premium principles. More precisely, we will prove that each premium principle satisfying Properties 1-5 belongs to Wang's class of premium principles. Also the inverse conclusion holds: Each premium principle belonging to Wang's class has Properties 1-5. The characterization of this class of premium principles is also considered in Wang, Young & Panjer (1997). Their approach is embedded in a slightly different setting. Also their definition of a family of risks for which the premium principle has to hold is different. They use a characterization theorem of Greco to prove their results. In our different setting, we are able to give a straightforward and simpler proof for the characterization of Wang's class of premium principles.

We obtain our results in different steps. First, we consider the case of risks with bounded support, which have a piecewise constant tail function. Next, we consider the case of general bounded risks. Finally, we prove our result for general risks.

Lemma 2 *Assume that a premium principle $H : \Gamma \rightarrow [0, \infty]$ has the Properties 1-4. Then there exists a unique distortion function g such that for all discrete risks $X \,\epsilon\, \Gamma$ with only finitely many mass points, we have that*

$$H(X) = \int_0^\infty g\left[S_X(x)\right] dx.$$

Furthermore, $g(q) \geq q$ for all $q \,\epsilon\, [0,1]$.

Proof. Consider a discrete risk X with finitely many mass points. Then there exist a positive integer n and sequences $0 = x_0 < x_1 < x_2 < ... < x_n$ and $1 \geq p_0 > p_1 > p_2 > ... > p_{n-1} > 0$ such that

$$S_X(x) = \sum_{i=0}^{n-1} p_i \; I\left(x_i \leq x < x_{i+1}\right), \qquad x \geq 0,$$

were $I\left(x_i \leq x < x_{i+1}\right)$ is the indicator function which equals 1 if $x_i \leq x < x_{i+1}$ and 0 otherwise.

We can write X as a sum of layers:

$$X = \sum_{i=0}^{n-1} L(x_i, x_{i+1})$$

It's easy to verify that the tail function of the layer $L(x_i, x_{i+1})$ is given by

$$S_{L(x_i,x_{i+1})}(x) = \begin{cases} p_i & : 0 \leq x < x_{i+1} - x_i \\ 0 & : x \geq x_{i+1} - x_i, \end{cases}$$

so that $L(x_i, x_{i+1})$ is a two-point distributed random variable with

$$\Pr\left[L(x_i, x_{i+1}) = x_{i+1} - x_i\right] = p_i = 1 - \Pr\left[L(x_i, x_{i+1}) = 0\right].$$

Now we will prove that the premium $H(X)$ can be written as

$$H(X) = \int_0^\infty g\left[S_X(x)\right] dx$$

with the distortion function g defined by

$$g(q) = H(B_q), \qquad 0 \leq q \leq 1.$$

Because of the property of additivity for comonotonic risks, we find

$$H(X) = \sum_{i=0}^{n-1} H\left(L(x_i, x_{i+1})\right)$$

Further, we have that $L(x_i, x_{i+1}) \stackrel{\mathcal{D}}{=} (x_{i+1} - x_i)\ B_{p_i}$ so that the scale-invariance property leads to

$$H\left(L(x_i, x_{i+1})\right) = (x_{i+1} - x_i)\ H(B_{p_i}) = (x_{i+1} - x_i)\ g(p_i).$$

Combining these results, we find

$$H(X) = \sum_{i=0}^{n-1} (x_{i+1} - x_i)\ g(p_i) = \sum_{i=0}^{n-1} \int_{x_i}^{x_{i+1}} g\left[S_X(x)\right] dx = \int_0^b g\left[S_X(x)\right] dx.$$

We can conclude that we have found a distortion function g, with $g(q) \geq q$ for all $q \ \epsilon\ [0,1]$, such that $H(X) = \int_0^\infty g\left[S_X(x)\right] dx$ for all discrete risks X with only finitely many mass points.

It easy to see that for any such distortion function we must have that $H(B_q) = g(q)$ for all $q \epsilon\, [0,1]$, which means that g is uniquely determined. ■

As the tail function of a risk X is not one-to-one, we have to be cautious in defining the inverse tail function. We will define the inverse tail function of a risk X as follows:

$$S_X^{-1}(q) = inf\left\{x : S_X(x) \leq q\right\}, \quad 0 \leq q < 1, \quad S_X^{-1}(1) = 0.$$

Now we are able to prove that if the inverse tail functions of two risks are close to each other, then also their respective premiums will be close to each other.

Lemma 3 *Consider a premium principle $H : \Gamma \rightarrow [0, \infty]$ satisfying Properties 1-3, and let X and Y be two risks in Γ. If there exists a constant $c \geq 0$ such that $\left|S_X^{-1}(q) - S_Y^{-1}(q)\right| \leq c$ for all $q \ \epsilon\ [0,1]$, then $|H(X) - H(Y)| \leq c$.*

Proof. The inequality $\left|S_X^{-1}(q) - S_Y^{-1}(q)\right| \leq c$ can be written as $S_Y^{-1}(q) - c \leq S_X^{-1}(q) \leq S_Y^{-1}(q) + c$. We will prove that if the right hand side inequality holds for all $q \in [0,1]$, then $H(X) \leq H(Y) + c$. The other part of the proof is similar.

From the right hand side inequality we find that $S_X^{-1}(q) \leq S_{Y+c}^{-1}(q)$ for all $q \in [0,1]$. This condition is equivalent to saying that $S_X(x) \leq S_{Y+c}(x)$ for all $x \geq 0$, see Dhaene, Wang, Young & Goovaerts (1997). The desired result is then obtained from Property 1 and the translation invariance of H. ■

The essential point of the proof in the lemma above, is the fact that X is stochastically dominated by $Y + c$. We can also prove this fact by using the technique of coupling: Let U be a random variable uniformly distributed on $(0,1)$. Then $S_X^{-1}(q) \leq S_Y^{-1}(q)+c$ for all $q \in [0,1]$ implies $S_X^{-1}(U) \leq S_Y^{-1}(U)+c$ with probability one. As a consequence, we find that $S_X^{-1}(U)$ is stochastically dominated by $S_Y^{-1}(U) + c$ which implies that X is stochastically dominated by $Y + c$. This elegant proof was mentioned to us by Müller, A.

In the following theorem, we use Lemma 3 to generalize Lemma 2 to the general case.

Theorem 4 *Assume that the premium principle $H : \Gamma \to [0,\infty]$ satisfies Properties 1-5. Then there exists a unique distortion function g, with $g(q) \geq q$ for all $q \in [0,1]$, such that for all risks $X \in \Gamma$ we have that*

$$H(X) = \int_0^\infty g\left[S_X(x)\right] dx.$$

Proof. First, we will assume that X has bounded support $[0,b]$. We can approximate $S_X(x)$ by the following piecewise constant tail function:

$$S_{X_n}(x) = \sum_{i=0}^{2^n-1} S_X\left(\frac{i+1}{2^n}b\right)\ I\left(\frac{i}{2^n}b \leq x < \frac{i+1}{2^n}b\right), \qquad x \geq 0.$$

It's easy to verify that $\left|S_X^{-1}(q) - S_{X_n}^{-1}(q)\right| \leq \frac{b}{2^n}$ for all $q \in [0,1]$. From Lemma 3 we then find that $|H(X) - H(X_n)| \leq \frac{b}{2^n}$. Hence, $H(X) = \lim_{n\to\infty} H(X_n)$. From Lemma 2 and the dominated convergence theorem, it follows that $H(X) = \int_0^b g\left[S_X(x)\right] dx$.

Now assume that X has an unbounded support. We have that for any $d \geq 0$, the risk $\min(X,d)$ is bounded with ddf given by

$$S_{\min(X,d)}(x) = \begin{cases} S_X(x) & : 0 \leq x < d \\ 0 & : x \geq d. \end{cases}$$

Hence, $H[\min(X,d)] = \int_0^d g[S_X(x)]\,dx$. The desired result then follows from Property 5. ■

Observe that also the inverse conclusion of Theorem 9 holds: Assume that the premium principle $H : \Gamma \to [0,\infty]$ is defined by

$$H(X) = \int_0^\infty g[S_X(x)]\,dx.$$

for some distortion function g, with $g(q) \geq q$ for all $q \in [0,1]$. Then this premium principles fulfils the Properties 1-5. A proof for the Properties 1-3 can be found in Wang (1996). The proof for Properties 4 and 5 is straightforward.

We finally remark that, within the framework of our theory, a premium principle is completely determined if the premiums for all Bernouilli risks are given. If the insurer wants to use a premium principle satisfying Properties 1-5, then it suffices to fix the premiums for all Bernouilli risks in order to solve the general problem of determining the premiums for all risks in the family of risks under consideration.

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