# Comonotonicity, Correlation Order and Premium Principles

Shaun Wang
University of Waterloo

Jan Dhaene
Katholieke Universiteit Leuven

**ABSTRACT**

In this paper, we investigate the notion of dependency between risks and its effect on the related stop-loss premiums. The concept of comonotonicity, being an extreme case of dependency, is discussed in detail. For the bivariate case, it is shown that, given the distributions of the individual risks, comonotonicity leads to maximal stop-loss premiums. Some properties of stop-loss order preserving premium principles are considered. A simple proof is given for the sub-additivity property of Wang's premium principle.

*Keywords:* Dependency, correlation order, comonotonicity, stop-loss premium, premium principle.

## 1 Introduction

In many situations, individual risks are correlated since they are subject to the same claim generating mechanism or are influenced by the same economic/physical environment. For instance, the individual risks of an earthquake risk portfolio which are located in the same geographic area are correlated since individual claims are contingent on the occurrence and severity of the same earthquake. As another example, consider a bond portfolio. Individual bond default experience may be conditionally independent for given market conditions. However, the underlying economic environment (e.g. interest rates) may affect all individual bonds in the market in a similar way.

In traditional risk theory, individual risks are usually assumed to be independent, mainly because the mathematics for correlated risks are less tractable. Consequently, the aggregate claims distribution and the stop-loss premiums are evaluated under the independence assumption.

Intuitively, with the presence of positive correlation, the law of large numbers will no longer hold and the aggregate risk may exhibit greater deviation than in the case of independent risks. Therefore, for a positively correlated risk portfolio, the independence assumption would probably under-estimate the stop-loss premiums.

One main theme of this paper is to investigate the effect of correlation on stop-loss premiums when the assumption of mutually independence of the individual risks no longer holds.

A standard way of modeling situations where two individual risks $X_i$ $(i = 1, 2)$ are subject to the same external mechanism is to use a secondary mixing distribution. The uncertainty about the external mechanism is then described by a structure parameter $z$, which is a realization of a random variable $Z$. The aggregate claims can then be seen as a two-stage process: First, the external parameter $Z = z$ is drawn from the distribution function $F_Z$ of $Z$; the claim amount of each individual risk $X_i$ is then obtained as a realization from the conditional distribution function $F_{X_i}(x_i \mid Z = z)$ of $X_i$.

In this paper, we will introduce a special type of such a mixing model, namely the case where the conditional claim amounts $X_i \mid Z = z$ are degenerate and non-decreasing functions of $z$. Such a model is in a sense an extreme form of a mixing model, as in this case the external parameter $Z = z$ completely determines the aggregate claims. Risks that can be modeled by such a mixing model are said to be *comonotonic.*

In Section 2 the concept of comonotonicity and its close relation with Fréchet bounds for bivariate distribution functions is considered. In Section 3 the relation between comonotonicity and some of the results in Dhaene and Goovaerts (1996) is explored. In Section 4 we discuss the behavior of some premium principles in case the risks involved are not mutually independent. Finally, in Section 5 a simple proof is given for the sub-additivity property of the class of premium principles introduced in Wang (1995, 1996).

# 2 Comonotonicity and Fréchet Bounds

For a risk $X$ (i.e. a non-negative real valued random variable with a finite mean), we denote the cumulative distribution function (cdf) by $F_X(x) = \Pr\{X \leq x\}$, $(0 \leq x)$. The inverse $F_X^{-1}$ is defined as $F_X^{-1}(q) = \inf\{x : F_X(x) \geq q\}$, $(0 < q < 1)$. For a pair of risks $(X, Y)$, the bivariate cdf is given by $F_{X,Y}(x, y) = \Pr\{X \leq x, Y \leq y\}$, $(0 \leq x, y)$.

The concept of *comonotonicity* was introduced by Schmeidler (1986) and Yaari (1987), see also Roëll (1987). It has since then played an important role in economic theories of decision under risk and uncertainty.

**Definition 1** *Two risks $X$ and $Y$ are said to be comonotonic if their bivariate cdf satisfies*

$$F_{X,Y}(x, y) = \min\left(F_X(x), F_Y(y)\right) \qquad \textit{for all } x, y \geq 0.$$

From the definition above we see that, in order to find the probability of the outcomes of two comonotonic risks $X$ and $Y$ being less than $x$ and $y$ respectively, one simply takes the probability of the least likely of these two events.

The following theorem gives an equivalent condition for comonotonicity. The notation $\stackrel{d}{=}$ is used to indicate that the (bivariate) random variables involved have the same cdf.

**Theorem 1** *Two risks $X$ and $Y$ are comonotonic if and only if there exists a random variable $Z$ and non-decreasing functions $u$ and $v$ on $R$ such that $(X, Y) \stackrel{d}{=} (u(Z), v(Z))$.*

**Proof.** First, assume that there exists a random variable $Z$ and non-decreasing functions $u$ and $v$ on $R$ such that $(X, Y) \stackrel{d}{=} (u(Z), v(Z))$.
We have that

$$F_{X,Y}(x, y) = \Pr\{u(Z) \leq x, v(Z) \leq y\} = \Pr\{Z \epsilon A, Z \epsilon B\}$$

where $A$ and $B$ are intervals of the form $[0, d]$ or $[0, d[$.
As $A \subseteq B$ or $B \subseteq A$, we find

$$\begin{aligned} F_{X,Y}(x, y) &= \Pr\{Z \epsilon A, Z \epsilon B\} = \min(\Pr\{Z \epsilon A\}, \Pr\{Z \epsilon B\}) = \min(\Pr\{X \leq x\}, \Pr\{Y \leq y\}) \\ &= \min(F_X(x), F_Y(y)) \end{aligned}$$

which proves the first part of the theorem.

To prove the other part, assume that $X$ and $Y$ are comonotonic risks. Remark that for any $q \epsilon (0, 1)$ and $x \geq 0$ we have that $F_X^{-1}(q) \leq x \Longleftrightarrow q \leq F_X(x)$, and similarly for $Y$. Now let $U$ be any uniformly distributed random variable on $(0, 1)$. Using the equivalence relation, it is easy to verify that $(X, Y) \stackrel{d}{=} (F_X^{-1}(U), F_Y^{-1}(U))$. Furthermore, $F_X^{-1}$ and $F_Y^{-1}$ are non-decreasing, so that the theorem is proved. ■

From the theorem above, we see that comonotonic risks can indeed be considered as "common monotonic". We remark that comonotonicity of two risks means that these risks are not able to compensate each other. Other characterizations of comonotonic risks can be found in Denneberg (1994).

The concept of comonotonicity is closely related to the following result, which is usually attributed to both Hoeffding (1940) and Fréchet (1951).

**Theorem 2** *The joint cdf $F_{X,Y}(x, y)$ of the risks $X$ and $Y$ is constrainted from above and below by*

$$\max(F_X(x) + F_Y(y) - 1, 0) \leq F_{X,Y}(x, y) \leq \min(F_X(x),\ F_Y(y)).$$

Let $R(F_X, F_Y)$ be the class of all bivariate distributed random variables with marginals $F_X$ and $F_Y$ respectively. The bounds in Theorem 2 hold for all $(X, Y)$ in $R(F_X, F_Y)$.

In order to show that the Fréchet bounds are reachable within the class of all risks with given marginal distributions, remark that for any random variable $U$ which is uniformly distributed on $(0,1)$, we have that $(F_X^{-1}(U), F_Y^{-1}(U)) \, \epsilon \, R(F_X, F_Y)$ and has a bivariate cdf given by the Fréchet upper bound. Similarly, we have that $(F_X^{-1}(U), F_Y^{-1}(1-U)) \, \epsilon \, R(F_X, F_Y)$ and has a bivariate cdf given by the Fréchet lower bound.

The concept of comonotonicity can be explained in terms of Monte Carlo simulation. We have seen that risks $X$ and $Y$ are comonotonic if and only if $(X,Y) \stackrel{d}{=} (F_X^{-1}(U), F_Y^{-1}(U))$, for $U$ being any uniformly distributed random variable on $(0,1)$. Hence, in order to simulate comonotonic risks, one needs to generate only one sample of random uniform numbers and insert in $F_X^{-1}$ and $F_Y^{-1}$ to get a sample of pairs of $(X,Y)$.
By contrast, if $X$ and $Y$ are independent, then one needs to generate two independent samples of random uniform numbers and then insert these two sets in $F_X^{-1}$ and $F_Y^{-1}$ respectively.

Recall that $X$ and $Y$ are (positively) perfectly correlated if and only if there exist real numbers $a > 0$ and $b$ such that $Y = aX + b$, except, perhaps, for values of $X$ with zero probability. It follows immediately that positive perfect correlation of $X$ and $Y$ implies $F_{X,Y}(x,y) = \min\left(F_X(x),\ F_Y(y)\right)$. Hence, comonotonicity is an extension of the concept of positive perfect correlation.
Consider e.g.

$$X_1 = \begin{cases} X, & X \leq d \\ d, & X > d, \end{cases} \qquad X_2 = \begin{cases} 0, & X \leq d \\ X - d, & X > d. \end{cases}$$

Then $X_1$ can be interpreted as the part of total claims to be covered by the primary insurer and $X_2$ the part to be covered by the stop-loss reinsurer. It follows that $X_1$ and $X_2$ are *not* perfectly correlated since one cannot be written as a function of the other. However, since $X_1$ and $X_2$ are non-decreasing functions of the original risk $X$, they are comonotonic.
More generally, we can say that most risk sharing schemes (between insurer and reinsurer, or between insured and insurer) lead to partial risks that are comonotonic. The only restriction that has to hold is that both risk sharing partners have to bear more (or at least as much) if the underlying total claims increase. An example of a risk sharing scheme which does not lead to partial comonotonic risks is a policy with a franchise deductible where the risk taken by the insured and the insurer are given by

$$X_1 = \begin{cases} X, & X \leq d \\ 0, & X > d, \end{cases} \qquad X_2 = \begin{cases} 0, & X \leq d \\ X, & X > d. \end{cases}$$

respectively. In this case a larger loss can be advantageous for the insured.

## 3 Stop-Loss Order and Correlation Order

For any risk $X$ and any $d \geq 0$, we define $(X-d)_+ = \max(0, X-d)$. The net stop-loss premium with retention $d$ is then defined by $\mathrm{E}(X-d)_+$.

**Definition 2** *A risk $X$ is said to precede a risk $Y$ in stop-loss order, written $X \leq_{sl} Y$, if for all retentions $d \geq 0$, the net stop-loss premium for risk $X$ is smaller than that for risk $Y$:*

$$\mathsf{E}(X-d)_+ \leq \mathsf{E}(Y-d)_+.$$

More details on this partial order between distribution functions can be found e.g. in Kaas *et al* (1994) and Müller (1996).

In Dhaene and Goovaerts (1996) ordering relations are investigated for the elements of the class $R(F_X, F_Y)$.

**Definition 3** *Let $(X_1, Y_1)$ and $(X_2, Y_2)$ be two elements of $R(F_X, F_Y)$. We say that $(X_1, Y_1)$ is less correlated than $(X_2, Y_2)$, written $(X_1, Y_1) \leq_{corr} (X_2, Y_2)$, if either of the following equivalent conditions holds:*
*(1) For all non-decreasing functions $f$ and $g$ for which the covariances exist,*

$$\mathsf{Cov}(f(X_1), g(Y_1)) \leq \mathsf{Cov}(f(X_2), g(Y_2)).$$

*(2) For all $x, y \geq 0$, the following inequality holds:*

$$F_{X_1,Y_1}(x,y) \leq F_{X_2,Y_2}(x,y).$$

The second condition implies that the probability that both random variables of a couple realize small (large) values is smaller for the least correlated couple.

In many situations, insurance risks tend to act "more similarly" than in the independent case. A useful concept of bivariate dependency which can be used in such situations is positive quadrant dependency.

**Definition 4** *The risks $X$ and $Y$ are said to be positively quadrant dependent, written $PQD(X,Y)$, if either of the following equivalent conditions holds:*
*(1) For all non-decreasing functions for which the covariances exist, we have that*

$$\mathsf{Cov}(f(X), g(Y)) \geq 0.$$

*(2) For all $x, y \geq 0$, the following inequality holds:*

$$F_{X,Y}(x,y) \geq F_X(x).F_Y(y).$$

It follows immediately that $PQD(X, Y)$ is equivalent to saying that $X$ and $Y$ are more correlated (in the sense of Definition 4) than if they were independent.

It is obvious that perfect positive correlation implies comonotonicity, which in turn implies positive quadrant dependency.

We can also introduce the notion of negative quadrant dependency, which is in a sense the opposite of positive quadrant dependency.

**Definition 5** *The risks $X$ and $Y$ are said to be negatively quadrant dependent, written $NQD(X, Y)$, if either of the equivalent conditions in Definition 5 holds, with $\geq$ replaced by $\leq$.*

The notion of "negative quadrant dependency" can be used to describe situations where insurance risks are less correlated than in the independent case. For a reference to the notions of PQD and NQD, see e.g. Barlow et al. (1975). For actuarial applications, see Norberg (1989) and Dhaene et al. (1997).

The following result (due to Dhaene and Goovaerts(1996)) connects correlation order between pairs $(X_i, Y_i)$, $(i = 1, 2)$ and stop-loss order between the corresponding sums $X_i + Y_i$.

**Theorem 3** *Let $(X_1, Y_1)$ and $(X_2, Y_2)$ be elements of $R(F_X, F_Y)$. If*

$$(X_1, Y_1) \leq_{corr} (X_2, Y_2)$$

*then*

$$X_1 + Y_1 \leq_{sl} X_2 + Y_2.$$

Theorem 3 states that correlation order between two couples of random variables with given marginal distribution functions implies a stop-loss order between their respective sums.

Combining Fréchet's result and Theorem 3, we find the following result.

**Theorem 4** *Let $U$ be uniformly distributed on $(0, 1)$. Then for any pair of risks $(X, Y)$ the following ordering relations hold:*
*(1) $(F_X^{-1}(U), F_Y^{-1}(1-U)) \leq_{corr} (X, Y) \leq_{corr} (F_X^{-1}(U), F_Y^{-1}(U))$*
*(2)$F_X^{-1}(U) + F_Y^{-1}(1-U) \leq_{sl} X + Y \leq_{sl} F_X^{-1}(U) + F_Y^{-1}(U)$.*

From Theorem 4 we see that the Fréchet upper (lower) bound yields the maximal (minimal) stop-loss premiums in the class of all bivariate distributions with given marginals:

$$\int_0^1 [F_X^{-1}(q) + F_Y^{-1}(1-q) - d]_+ \, dq \leq \mathsf{E}(X + Y - d)_+ \leq \int_0^1 [F_X^{-1}(q) + F_Y^{-1}(q) - d]_+ \, dq.$$

Hence, for any pair of risks $(X, Y)$, we have found an upper and a lower bound for the

stop-loss premiums of $X+Y$. These bounds are expressed in terms of the (inverse) cdf's of $X$ and $Y$. Hence, they hold for any pair of random variables contained in $R(F_X, F_Y)$, regardless of their dependency structure. For an expression of the upper bound in case of exponential marginals, see Heilmann (1986).
In the limiting case of full reinsurance, i.e. $d = 0$, the lower and the upper bound both reduce to $\int_0^1 [F_X^{-1}(q) + F_Y^{-1}(q)]dq$ which is equal to E$X$+E$Y$.

# 4 Premium Principles

The net premium of a risk $X$ is defined as the expectation of $X$. Insurers usually charge a risk-adjusted premium, being the sum of the net premium and some risk load. A premium principle $\pi$ is a mapping that assigns to any risk $X$ a positive value $\pi(X)$, which is called the risk-adjusted premium. We will assume that risks with the same cdf lead to the same risk-adjusted premium.
A desirable property for a premium principle is that it preserves stop-loss order, i.e. $X \leq_{sl} Y$ implies that $\pi(X) \leq \pi(Y)$ (see e.g. Kaas et al, 1994).
From Theorem 3 we immediately find the following result:

**Theorem 5** *Let $\pi$ be a premium principle which preserves stop-loss order, and $(X_1, Y_1)$ and $(X_2, Y_2)$ be elements of $R(F_X, F_Y)$. If*

$$(X_1, Y_1) \leq_{corr} (X_2, Y_2)$$

*then*

$$\pi(X_1 + Y_1) \leq \pi(X_2 + Y_2).$$

From the Theorems 4 and 5, we find the following corollary.

**Corollary 6** *Let $\pi$ be a premium principle which preserves stop-loss order. Then we have*

$$\pi\left(F_X^{-1}(U) + F_Y^{-1}(1-U)\right) \leq \pi(X+Y) \leq \pi\left(F_X^{-1}(U) + F_Y^{-1}(U)\right).$$

Corollary 6 states that for a stop-loss preserving premium principle, the risk-adjusted premium of a sum of two risks is maximal if the two risks are comonotonic. As comonotonic risks can be considered as bets on the same event, neither of them is a hedge against the other. So it seems a desirable property that the premium of the sum is maximal in this case.
On the other side, we see that the premium of $X+Y$ is minimal if $(X, Y) \stackrel{d}{=} \left(F_X^{-1}(U), F_Y^{-1}(1-U)\right)$. In this case the combination of both risks leads to an optimal hedge as the higher the one risk, the lower the other one will be. So it seems to be a desirable property that the lowest risk-adjusted premium is obtained in this case.

A premium principle is called additive within a given class of risks if the premium for the sum of any two risks taken from this class equals the sum of the individual premiums. A premium principle is said to be sub-additive (super-additive) if the premium for the sum is not larger (not smaller) than the sum of the individual premiums. From Theorem 5 we find the following result.

**Corollary 7** *If a premium principle preserves stop-loss order and is additive for independent risks, then it is sub-additive for negative quadrant dependent risks, and super-additive for positive quadrant dependent risks:*

$$\pi(X+Y) \leq \pi(X) + \pi(Y) \quad \textit{if } NQD(X,Y);$$

$$\pi(X+Y) \geq \pi(X) + \pi(Y) \quad \textit{if } PQD(X,Y).$$

As a special case of Corollary 7, we find that a stop-loss order preserving premium principle which is additive for independent risks, is super-additive for comonotonic risks.
Remark that the well-known exponential premium principle satisfies the conditions of Corollary 7, see e.g. Kaas et al. (1994).

In the following corollary, we consider the case that the premium principle is additive for comonotonic risks.

**Corollary 8** *If a premium principle preserves stop-loss order and is additive for comonotonic risks, then it is sub-additive:*

$$\pi(X+Y) \leq \pi(X) + \pi(Y) \quad \textit{for all risks } X \textit{ and } Y.$$

Hence, premium principles which satisfy the conditions of Corollary 8 always give a volume discount.

Remark that a comonotonic-additive premium principle is scale-invariant, i.e. $\pi(aX) = a\pi(X)$ for all $a > 0$. The opposite conclusion is not true in general: the exponential premium principle is scale-invariant, but not comonotonic-additive.

The question which kind of conditions (the one from Corollary 7 or 8) are preferable, depends upon the situation under consideration.
Consider two pairs of risks $(X, Y)$ and $(X', Y')$ with the same marginal cdf's. Assume that $X$ and $Y$ are comonotonic parts of one combined risk $X+Y$. The premium for this combined risk equals $\pi(X+Y)$. Further, assume that $X'$ and $Y'$ are the risks belonging to two different policyholders. These policyholders pay a total premium equal to $\pi(X)+\pi(Y)$ which is independent of the correlation structure between the two risks. The insurer will prefer the risks $X'$ and $Y'$ over $X$ and $Y$ because comonotonic risks are bets on the same event. The insurer can incorporate this preference in his premium structure by choosing a

premium principle that is super-additive for such risks, i.e. $\pi(X+Y) \geq \pi(X) + \pi(Y)$ for all $X$ and $Y$ that are comonotonic. This means that the insurer is not willing to give a reduction in the risk load for a combined policy of comonotonic risks.
Next, assume that each policyholder is free to split his risk and buy separate policies for this split risks from the same insurer. In this situation , the insurer should use a sub-additive premium principle, i.e. $\pi(X+Y) \leq \pi(X) + \pi(Y)$ for all risks $X$ and $Y$, since otherwise the policyholder will be better off by buying separate policies.
We can conclude that if the insurer is not willing to give a reduction for a combined policy of comonotone risks and if he wants to avoid splitting of risks (assumed that it is possible), then he should use a premium principle that is additive for comonotonic risks.
Remark that if splitting of risks is not possible, then the insurer can use a premium principle that is super-additive for comonotonic risks. An example is catastrophe insurance, where the insurer will only be willing to insure a larger part of the complete risk at a higher risk load.

Wang (1996) proposes to compute the risk-adjusted premiums by the following premium principle:

$$H_g[X] = \int_0^\infty g[1 - F_X(x)]dx = \int_0^1 F_X^{-1}(1-q)dg(q),$$

where $g$ is a non-decreasing concave function with $g(0) = 0$ and $g(1) = 1$.
Note that for $g(x) = x$ $(0 \leq x \leq 1)$, $H_g[X] = \mathsf{E}X$.

The interpretation of this class of premium principles is clear: First, the original tail function $1 - F_X(x)$ of the risk $X$ is replaced by a new tail function $g(1 - F_X(x))$ which gives more weight to the right-tail. Then the risk adjusted premium is computed as the expectation of $X$ under the new tail function.
Wang's premium principle preserves some common ordering of risks such as stop-loss ordering.

**Theorem 9** *Wang's premium principle preserves stop-loss order, i.e.*

$$X \leq_{sl} Y \Longrightarrow H_g[X] \leq H_g[Y].$$

*Moreover, it is additive in the class of comonotonic risks,*

$$H_g[X+Y] = H_g[X] + H_g[Y] \quad \textit{for comonotonic risks } X \textit{ and } Y.$$

**Proof.** See Wang (1996). ■

It can be shown that transforming the tail as is done in Wang's premium principle, is the only way to get comonotonic-additive and stop-loss order preserving premium principles. Hence, outside Wang's class of premium principles there are no premium principles that have these two properties simultaneously, see Denneberg (1994).

## 5 A Simple Proof of Sub-additivity

In Wang (1995), a proof (due to Ole Hesselager) is given for the sub-additivity property of Wang's premium principle in the special case where $g(x)$ is of the form $g(x) = x^c, 0 < c < 1$. It was stated in Wang (1996) that the proof for this special case could readily be generalized to other functions $g$. However, Denneberg pointed out (via personal communication) that this statement is not true. As an application of the present paper, now we can give a correct and simple proof for the sub-additivity theorem.

**Theorem 10** *For any two risks, regardless of their dependency relation, we have that*

$$H_g[X+Y] \leq H_g[X] + H_g[Y].$$

**Proof.** The proof follows immediately from Corollary 8 and Theorem 9. ■

Remark that the upper bound in Theorem 10 corresponds to the case that both risks are comonotonic.

From Corollary 6 and Theorem 9 we also find the following lower bound for $H_g[X+Y]$:

$$H_g[X+Y] \geq H_g[F_X^{-1}(U) + F_Y^{-1}(1-U)].$$

The lower bound corresponds to the case that both risks are maximum hedges against each other.
Finally, remark that if $X$ and $Y$ are positive quadrant dependent, then a better lower bound is given by $H_g\left[X^{ind} + Y^{ind}\right]$ where $X^{ind}$ and $Y^{ind}$ are mutually independent and have the same marginal cdf's as $X$ and $Y$ respectively.

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# Correspondence

Shaun Wang, Dept. of Statistics and Actuarial Science, University of Waterloo, ON N2L 3G1, Canada (sswang@math.uwaterloo.ca).

Jan Dhaene, Dept. Toegepaste Economische Wetenschappen, K.U. Leuven, Huis Eighen Heerd, 3000 Leuven, Belgium, (Jan.Dhaene@econ.kuleuven.ac.be).