A liability driven approach to asset allocation

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Abstract.

We investigate a liability driven methodology for determining optimal asset mixes. We study the effect on the optimal investment strategy when changing the duration of the liability cash flow stream, changing the certainty level and changing the correlation matrix. It is shown that the methodology leads to results which are in accordance with intuition.

Keywords: Liability Driven Investing, Strategic Asset Allocation

1 INTRODUCTION

In this paper we determine optimal investment strategies in a liability driven environment. Starting from a given liability cash flow stream, we determine the optimal amount needed to meet these liabilities, as well as the related optimal investment strategy for this amount. The optimal investment strategy is called 'liability driven' in the sense that the assets are managed relative to the liabilities, as opposed to, for example, a strategy where one tries to outperform a given benchmark. The methodology is described in detail in Dhaene, Vanduffel, Goovaerts, Kaas & Vyncke (2005)[4].

In Section 2 we investigate the sensitivity of optimal investment strategies with respect to changes in the duration of the liabilities, changes in the required certainty level and changes in the correlation structure of the underlying asset classes. Some frequently asked questions are considered in Section 3. In Section 4 we consider a realistic example. Section 5 concludes the paper. We will assume that the return process of the available asset classes is modeled by a multivariate geometric Brownian motion process. The optimal investment strategy is chosen from the class of constant mix strategies.

2 OPTIMAL INVESTMENT STRATEGIES

Throughout Section 2, we will assume that the following asset classes are available: equity, real estate, bonds and cash. Their respective (yearly) drifts and volatilities are given in Table 1.

	μ	σ
Equity	9,00%	18,00%
Real Estate	7,00%	10,00%
Bonds	5,00%	6,00%
Cash	2,00%	1,50%

Table 1: Drifts and volatilities.

The correlation matrix describing the dependencies between the different asset class returns is given in Table 2.

	Equity	Real Estate	Bonds	Cash
Equity	100%	50%	20%	3%
Real Estate		100%	20%	- 10%
Bonds			100%	- 30%
Cash				100%

Table 2: Correlation matrix.

The time unit is chosen to be equal to 1 year. Let time 0 denote the present time. We will consider two scenarios for the liability cash flow stream. Scenario 1 refers to a single cash flow consisting of a liability payment of 1.242.381 at time 8. Scenario 2 refers to a series of 3 cash flows: a payment of 621.190 at time 8, a payment of 385.877 at time 16 and finally, a payment of 479.407 at time 24.

Starting from a given liability cash flow stream, the method described in Dhaene et al. (2005) allows one to determine the optimal amount needed to cover these liabilities, as well as the related optimal investment strategy for this amount. Therefore, for each admitted investment strategy, one considers the stochastic provision. This stochastic provision is defined as the stochastically discounted value of all future liability payments, where discounting is performed using the stochastic return process of the investment strategy under consideration.

We will call the optimal amount needed to cover these liabilities the provision. However it is important to note that depending on the application at hand, this optimal amount could also be interpreted as the total amount of required assets, being the sum of provisions and required additional capital.

For a given cash flow stream, the optimal investment strategy (or asset mix), at a given certainty level p, $0<p<1$, is defined as the

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[4] Calculations were made with the software VACS-ALM, in which the method described in Dhaene et al. (2005) is implemented.

constant mix strategy that minimizes the VaR of the stochastic provision at level p. This is the total minimum amount that is needed by the company to guarantee, when invested according to the optimal strategy, a ruin probability of at most (1-p).

In Table 3 and Figure 4, the optimal investment strategies for both scenarios, at a certainty level of 95%, are presented.

	Equity	Real Estate	Bonds	Cash	Provision	Expected Return	Volatility
Sc. 1	5,37%	26,11%	49,06%	19,45%	1.035.530	5,15%	4,75%
Sc. 2	11,12%	36,68%	52,20%	0,00%	907.699	6,18%	6,46%

Table 3: Optimal asset mix for the two scenarios.

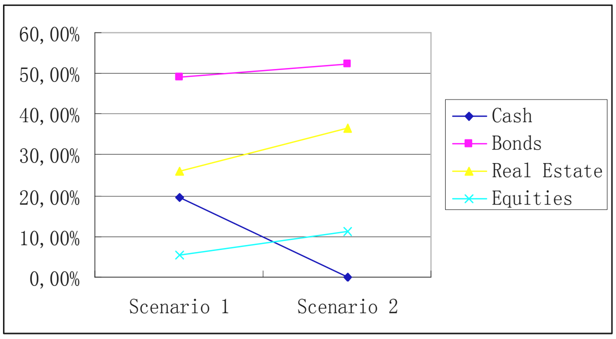

Figure 4: Optimal asset mix for the two scenarios.

From Table 3 and Figure 4, we can conclude that the optimal investment strategy strongly depends on the cash flow pattern. To be more specific, scenario 2 leads to a less conservative investment strategy than scenario 1. Indeed, for the second scenario, the proportions to be invested in equity, real estate and bonds all increase, whereas the proportion invested in cash is reduced to 0.

Intuitively, this move towards a more risky investment strategy could be expected because the second liability cash flow stream has a much longer duration, which allows a more pronounced time diversification effect. Also notice that the more risky investment strategy for the second scenario leads to a higher μ and σ.

Next, we restrict to scenario 2 and determine optimal investment strategies corresponding with different certainty levels.

	85%	90%	95%	99%
Equity	19,71%	15,16%	11,12%	6,97%
Real Estate	45,30%	40,74%	36,68%	31,84%
Bonds	34,99%	44,10%	52,20%	58,40%
Cash	0,00%	0,00%	0,00%	2,79%
Provision	795.022	841.021	907.699	1.036.882
Expected Return	6,69%	6,42%	6,18%	5,83%
Volatility	7,77%	7,03%	6,46%	5,84%

Table 5: Optimal asset mix for different certainty levels, scenario 2.

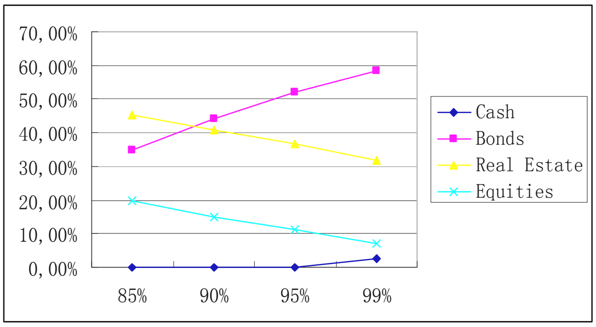

Figure 6: Optimal asset mix for different certainty levels, scenario 2.

From Figure 6, we can conclude that increasing the certainty level leads to a more conservative optimal investment strategy. The optimal investment strategy becomes more conservative by decreasing the proportions invested in equity and in real estate. As long as the certainty level is not too high, the investment strategy is made more conservative by additionally increasing the proportion invested in bonds, while keeping the proportion invested in cash equal to 0%. If the certainty level becomes sufficiently high, the investment strategy can only be made more conservative by not only investing more in bonds, but also investing in cash. From Table 5, we also see that increasing the certainty level does not only lead to a lower σ but also to a lower μ and a higher initial provision.

The obtained results have an intuitive interpretation: requiring a lower ruin probability leads to a more conservative investment strategy and a higher provision. Avoiding risk has a cost.

Finally, we investigate the influence of the correlations on the optimal asset mix. Therefore, we consider the following correlation matrix, of which all correlations are higher than the corresponding correlations in the original matrix:

	Equity	Real Estate	Bonds	Cash
Equity	100%	99%	40%	6%
Real Estate		100%	40%	- 5%
Bonds			100%	- 15%
Cash				100%

Table 7: Correlations.

Note that all correlations are higher than the corresponding correlations in the original matrix.

In Table 8 and Figure 9, we consider scenario 2 and compare the optimal investment strategies for both correlation structures. At the 95% - certainty level, we find the following results:

	Equities	Real Estate	Bonds	Cash	Provision	Expected Return	Volatility
Old Corr.	11,12%	36,68%	52,20%	0,00%	907.698	6,18%	6,46%
New Corr.	0,00%	45,33%	54,67%	0,00%	947.509	5,91%	6,57%

Table 8: Comparison of different correlations, p=95%.

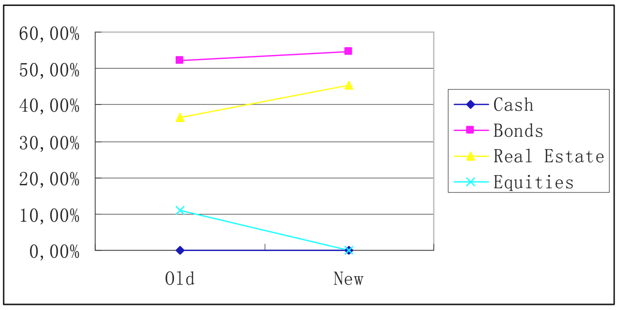

Figure 9: Comparison of different correlations, p=95%.

We can conclude that higher correlations lead to an increase in the proportions invested in real estate and bonds, at the cost of a decrease in the proportion invested in equity. This means that the investment corresponding with the highest correlations is the most conservative.

This shift in optimal proportions could be expected, as lower correlations lead to a higher asset diversification effect and vice versa. More asset diversification allows one to invest more in risky assets, which leads to a higher return without increasing the volatility of the investment. This is also reflected in the lower initial provision.

Hence the investor will prefer asset classes which are less correlated, in order to be able to benefit optimally from the asset diversification effect.

This diversification effect is even more prominent for a probability level of 99%, as is shown in Table 10 and Figure 11.

	Equities	Real Estate	Bonds	Cash	Provision	Expected Return	Volatility
Old Corr.	6,97%	31,84%	58,40%	2,79%	1.036.882	5,83%	5,84%
New Corr.	0,00%	26,34%	44,82%	28,84%	1.081.032	4,66%	4,42%

Table 10: Comparison of different correlations, p=99%.

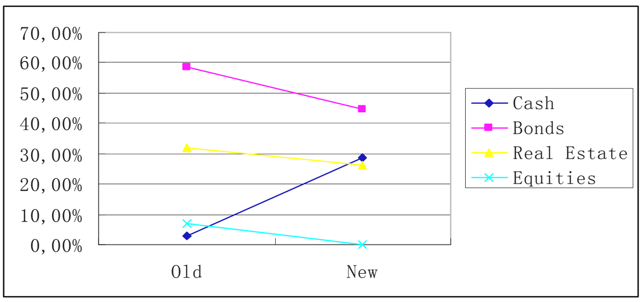

Figure 11: Comparison of different correlations, p=99%.

In this case, increasing the correlations leads to lower proportions invested in bonds, real estate and equity, while increasing the proportion invested in cash.

3 FREQUENTLY ASKED QUESTIONS

In this section, we illustrate how to (and how not to) apply the optimal allocation methodology described in Dhaene et al. (2005) for solving strategic asset allocation problems.

Throughout this section, we assume that the following asset classes are available: government bonds, corporate bonds and equity. The respective parameters μ and σ are given in Table 12.

Asset class	Type	μ	σ
Government Bonds	Belgium (BGB)	3,44%	1,83%
	Switzerland (SGB)	4,02%	0,82%
Corporate Bonds	U.S. (UCB)	3,34%	2,69%
	Europe (ECB)	3,52%	3,00%
Equity	ABC	6,37%	12,52%
	Eurostoxx	6,35%	10,65%

Table 12: Drifts and volatilities.

The correlation matrix describing the dependencies between the different asset class returns is given in the following Table:

	BGB	SGB	UCB	ECB	ABC	Eurostoxx
BGB	100%	95%	90%	90%	-10%	-20%
SGB		100%	90%	90%	-10%	-20%
UCB			100%	95%	-15%	-25%
ECB				100%	-15%	-25%
ABC					100%	95%
Eurostoxx						100%

Table 13: Correlations.

In Table 14, the optimal investment strategies for the two scenarios, at a certainty level of 99%, are presented:

	BGB	SGB	UCB	ECB	ABC	Eurostoxx	Certainty level
Sc. 1	0,00%	95,99%	0,00%	0,00%	0,00%	4,01%	99%
Sc. 2	0,00%	94,91%	0,00%	0,00%	0,00%	5,09%	99%

Table 14: Optimal investment strategies, p=99%.

Why are the two optimal asset allocations very conservative?

The choice of a required survival probability of 99% over the 8 year period might be a good figure from the point of view of the regulator, but this may not be the case from a management point of view: In a going-concern perspective, management may perhaps focus more on the 'risk around the mean'. This could be achieved by choosing a much lower probability level.

As $(99{,}88\%)^8 = 99\%$, one can say that the 8-year certainty level of 99% corresponds with yearly survival probabilities of 99,88%. Similarly, the 24-year certainty level of 99% corresponds with yearly survival probabilities of 99,96%.

Hence the use of a 99% certainty level in the application may be an overly strict requirement and will lead to very conservative optimal investment strategies, as can be seen from the proportions in Table 14.

Why is the optimal asset mix almost identical for all scenarios?

Concerning the choice of the admissible investment instruments, observe that in the class of bonds, the Swiss government bond dominates the 3 other bonds in a Markowitz-sense (highest μ and lowest σ). Moreover, all bond returns are highly positively correlated (i.e. almost comonotonic). Hence, investing in different bonds has almost no diversification effect.

Concerning the investment possibilities in equity, a similar remark can be made: the returns of Eurostoxx dominate the returns of ABC, and both returns are highly dependent as well. From these observations, together with the high value of the certainty level, we can conclude that any rational decision-maker will mainly invest in the Swiss government bond class.

Any ALM procedure that would lead to another investment decision is highly suspicious. Note that this observation is due to input, not to methodology.

What is an appropriate certainty level?

In general, it is impossible to compare the scenarios if the same certainty level of 99% is used for scenario 1 (8 years) and scenario 2 (24 years). In order to be able to compare the results for the two scenarios, a certainty level of 97% for scenario 2 would have been more appropriate. Indeed, a safety level of 99% for 8 years is roughly equivalent to a certainty level of 0.99 x 0.99 x 0.99 = 0.97 over the 24 year period.

Hereafter, we show the optimal investment strategy for each of the two scenarios for different certainty levels, which correspond (approximately) to a yearly certainty level of 99,5%:

	BGB	SGB	UCB	ECB	ABC	Eurostoxx	Certainty level
Sc. 1	0,00%	95,29%	0,00%	0,00%	0,00%	4,71%	96%
Sc. 2	0,00%	91,20%	0,00%	0,00%	0,00%	8,80%	90%

Table 15: Optimal investment strategies, yearly certainty level of 99,5%.

Is the choice of a multivariate geometric Brownian motion always appropriate for modeling the asset class returns?

Our methodology can be used to determine optimal investment strategies in the sense that the optimal proportions to be invested in a number of given asset classes (or investment accounts) are calculated. In order to do so, each asset class is specified by the parameters μ and σ of its yearly returns and also by the correlations of its yearly returns with the yearly returns of the other asset classes.

On the other hand, our method cannot always be used to appoint individual assets in the optimal portfolio. In particular, it cannot be used to allocate individual bonds as being optimal.

The evolution of the price of an asset can only be described by a geometric Brownian motion process in case the price of this asset is more uncertain, the further the future evaluation date. In this sense, an individual bond price (e.g. the one of the Swiss government bond GBG 4,5 2037) can never be described by a geometric Brownian motion process. Indeed, the Swiss government bond price will converge (with certainty) to its face value when approaching the expiration date.

Our model can be applied to a 'portfolio of bonds of a certain type,' specified by its μ and σ, and also by its correlations with the other asset returns. An example of an asset class is 'Belgian 10 year government bonds'. The μ and σ of this class reflect the expected return and volatility in the long run of 'Belgian 10 year government bonds'. These parameters will be driven by the duration of the bonds involved.

Theoretical evidence, but also empirical data, indicates that the lognormal assumption adequately fits the return pattern of a portfolio of bonds of a certain type.

After having obtained the proportions to be invested in each asset class, the choice of which assets belonging to this class have to be purchased is a problem that has to be solved by the investor, taking into account the duration of the liabilities.

It is important to note that in the case that a bond is held until its expiration date, the cash flow of liabilities has to be adjusted accordingly. So from a technical point of view we can also consider an extra asset class of bonds that are held until maturity.

As the time unit that we consider is long (typically 1 year), assuming a Gaussian model seems to be appropriate, at least approximately, by the Central Limit Theorem. In order to verify whether this theoretical setup can be compared with the data generating mechanism of real situations, we refer to Cesari & Cremonini (2003) and Lévy (2004). The first authors investigate four well-known stock market indices in US dollars, from Morgan Stanley: MSCI World, North America, Europe and Pacific, covering major stock markets in industrial as well as emerging countries. For the period 1997-1999, the authors conclude that daily returns are indeed both non-normal and auto-correlated. For monthly (and longer) periods however, they conclude that normal and independent returns will emerge.

Does the methodology takes into account the existing investment portfolio?

The existing investment portfolio (the proportions invested in the different asset classes) can be taken into account by putting constraints on the proportions. For example suppose one has invested 12% in Swiss government bonds and one is searching for the optimal investment strategy, without having to change the entire investment portfolio. This can be done by imposing the constraint that the proportion invested in Swiss government bonds lies in the range between 8% and 16%.

In the following example, we assume that we constraint Belgian Government bonds (to a maximum of 30%) and Swiss Government Bonds (to a maximum of 20%). Then we see that the proportions for scenarios 1 and 2 will not be similar anymore. Note that we use a 99% certainty level for scenario 1 and a 97% certainty level for scenario 2:

Sc. 1	Name	Mix
Government Bonds	BGB	30,00%
	SGB	20,00%
Corporate Bonds	UCB	19,07%
	ECB	17,91%
Equity	ABC	0,00%
	Eurostoxx	13,01%

Table 16: Optimal asset mix, p=99%.

Sc. 2	Name	Mix
Government Bonds	BGB	30,00%
	SGB	20,00%
Corporate Bonds	UCB	0,00%
	ECB	31,66%
Equity	ABC	0,00%
	Eurostoxx	18,34%

Table 17: Optimal asset mix, p=97%.

We observe that in this case, scenario 2 leads to a slightly more risky optimal investment strategy. Indeed, scenario 2 invests more in the more risky corporate bond (ECB is more risky than UCB) and also more in equity. Note that there is no investment in ABC, since Eurostoxx and ABC have almost the same expected return, while the former is less risky.

Finally, we remark that it may also be useful to have a look at the optimal investment strategy for covering future liabilities, without taking into account the current investment portfolio. Comparing this optimal portfolio with the existing portfolio will give an idea of the lost opportunities by not following the optimal investment strategy.

Is the insurer on the safe side in the case that the actual provision is higher than the optimal provision?

No, in general it is not true that in the case that the actual provision is higher than the optimal provision, the insurer is on the safe side. The reason why it is not true is that a given provision can never be evaluated on its appropriateness for covering the liabilities without knowing the related investment strategy of the underlying assets.

Hence it is possible that the insurer has a higher provision than is optimal, and nevertheless has a higher non-survival probability than with the lower optimal provision. This will be the case if the insurer is investing its assets in 'the wrong way'.

4. A REAL LIFE EXAMPLE

In this section, we consider the following real life liability cash flow stream of a portfolio of life annuities. All payments beyond year 2029 are aggregated in one figure at year 2029.

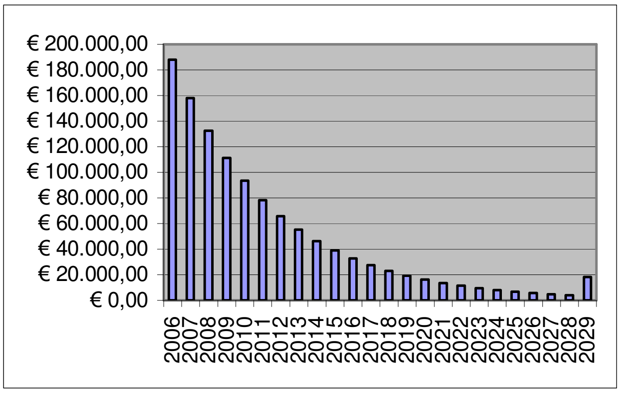

Figure 18: Liabilities, real life example.

Assume that the available asset classes and their parameters are given in Tables 1 and 2.

At a probability level of 90%, we find the following optimal asset mix:

	Equities	Real Estate	Bonds	Cash	Provision	Expected Return	Volatility
Optimal prop.	9,76%	35,32 %	54,93%	0,00%	975.093	6,10%	6,29%

Table 19: Optimal asset mix, without constraints.

We find that a large proportion is invested in real estate. This is due to the relatively high expected return compared to the relatively small volatility for this asset class. In practice, this high proportion invested in property will often be restricted. Therefore, we now determine the optimal investment strategy at a probability level of 90%, but with a proportion invested in real estate of at most 15%.

Under this constraint, we find the following optimal asset allocation:

	Equities	Real Estate	Bonds	Cash	Provision	Expected Return	Volatility
Optimal prop.	15,60%	15,00 %	69,40%	0,00%	982.145	5,92%	6,23%

Table 20: Optimal asset mix, proportion invested in real estate at most 15%.

In this case, the proportions invested in (the more risky) equities and (the less risky) bonds are increased. This results in a decreased expected return, and a slightly decreased volatility.

5. CONCLUSION

We investigated the liability driven methodology for determining optimal asset mixes as described in Dhaene, Vanduffel, Goovaerts, Kaas & Vyncke (2005). We studied the effect on the optimal investment strategy when changing the duration of the liability cash flow stream, changing the certainty level and changing the correlation matrix. Furthermore, we answered several frequently asked questions concerning the optimal strategy. It turns out that the methodology leads to results which are in accordance with intuition.

Finally notice that we illustrated the methodology by using VaR-based provisions. However, the method also allows to use a TailVaR based approach.

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