

Wearing a face mask: the new dress code?

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Over the last weeks, the debate on whether or not we should wear a face mask to fight the coronavirus has caused confusion. The World Health Organization continued to recommend to not wear a mask if you are not infected or not caring for someone who is sick. On the other hand, several countries, including Czech Republic, Israel, Morocco and Singapore, opted already in an early stage of the Covid-19 outbreak for compulsory face masks in public. In many other countries where wearing a mask is not mandatory, it is highly recommended by governments and health experts.

Why is this ‘new dress code’ not the norm across the world? Nassim Taleb, the famous author from ‘The Black Swan’, has strongly argued in favor of wearing masks from the early breakout of the virus as one of the key elements in a general strategy to prevent its spreading. His precautionary approach compares the possible costs of having made the wrong decision for each of the two alternatives. If we all wear masks, the worst that could happen is that masks turn out not to be helpful and that everybody experiences some minor inconveniences caused by having to wear it. On the other hand, if we do not wear masks, and one day it turns out that they were helpful, we would have missed a chance to fight the virus more effectively.

Comparing both consequences, a small inconvenience versus missing an opportunity to faster stop the virus, it becomes obvious that we should wear masks. In this note, we show that some basic mathematical considerations provide a simple answer to the question: everyone should be wearing a mask!

Suppose that each person carrying Covid-19 infects R other persons. This number R is the so-called *basic reproduction number* in epidemiology. In case this reproduction number R equals 2, a single person will infect two others and these 2 infected persons will again each infect two others. As a consequence, the number of infections at the second step is given by 4. A third round, where each of the 4 infected persons of the second step infects again 2 others, leads to 8 new infections. Suppose that this mechanism of each infected person infecting 2 others happens 10 times. After a chain of 10 infection rounds, the number of infected people in the tenth round is equal to $2 \times 2 \times 2 \times \dots \times 2$, a product involving 10 terms. Mathematicians denote this quantity by 2^{10} .

For an ‘ordinary’ flu, with a typical value of R equal to 1.3, an individual carrying the virus will infect on average 1.3 persons. If we apply the reasoning above, then at the last step of 10

rounds of infections, 14 persons are infected. But if R is equal to 2 (which is closer to the reproduction number of the recent coronavirus), then at step 10, the number of infected people equals 1 024. A huge difference!

Although epidemics with reproduction numbers 1.3 and 2 might at first sight look similar to each other, the latter one is vastly a much more severe disease. Even a relatively small difference in the reproduction number will have a huge impact on the total number of infections after a sufficient number of consecutive rounds of transmission. When a vaccine is not available, the problem of the exponentially growing number of transmissions can quickly turn from severe to devastating. In such cases, it is crucial to try to halt the future transmissions by introducing a strict policy in order to reduce the reproduction number. Such a policy may include precautionary measures such as social distancing and wearing face masks.

An argument often raised against the wear-a-mask policy is that individuals may tend to behave less careful under safety precautions. This behavioral phenomenon is known as the Peltzman effect. In particular, mandating or even only strongly advising protective masks may come at the expense of other precautionary advices, such as social distancing or avoiding face touching. In order to address these concerns, it is important to evaluate the extent to which the Peltzman effect offsets the benefits of wearing a mask. This amounts to evaluating the net decrease of the reproduction number after taking into account the potential offsetting factors. In order to reduce the impact of the Peltzman effect, it is very important that a wear-a-mask policy is accompanied by an appropriate educative support. As long as this effect does not completely 'eat away' the benefits of wearing a mask, it remains preferable to stick to this policy.

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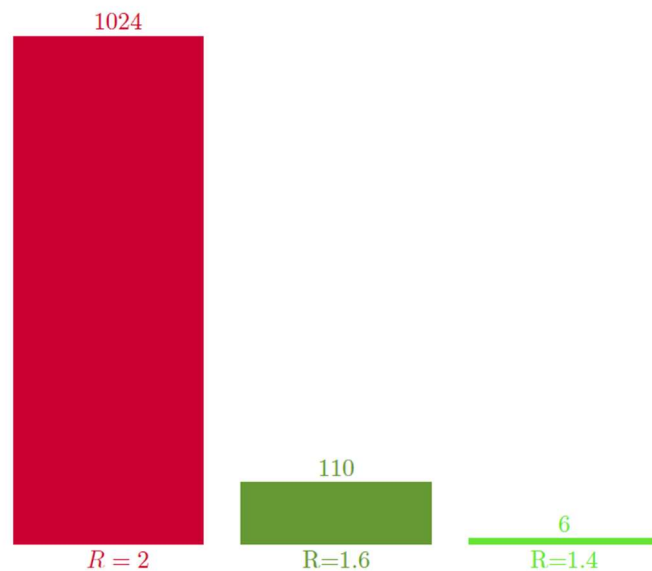
In order to illustrate the positive impact of wearing face masks, suppose that homemade masks become mandatory in public spaces, and that this policy reduces the reproduction number R by 20% from 2 to 1.6. The reduction percentage of 20% is a prudent choice, which attempts to account for the Peltzman effect. At the last step in a chain of 10 rounds of infections with reproduction

number R equal to 1.6, the number of infected people under the wear-a-mask policy now becomes $1.6 \times 1.6 \times \dots \times 1.6$, which is again a product of 10 terms, in mathematical notation $(1.6)^{10}$. In our example, wearing homemade masks leads to a number of 110 infected people in the tenth round, which is only 11% of the number of people that would have been infected without introducing that policy.

Instead of a 20% reduction of the original reproduction number, let us now look at what happens in case of a 40% reduction, from $R = 2$ to $R = 1.2$. This higher efficiency in the fight against the virus could be reached for instance by providing people with professional masks for use in public spaces. In this case, we find that after 10 steps the number of infected people

becomes 6. Expressed as a percentage, wearing professional masks reduces the number of infections in the tenth step to 0.6% of the number of infections without a wear-a-mask policy.

An important phenomenon becomes apparent when comparing the two situations we considered. The first situation was a 20% reduction of the reproduction number. The second one was a 40% reduction. As can be observed from the figure, the first 20% reduction is much more efficient than the second one. Indeed, after 10 rounds, we found a decrease of the number of infections by 914 for the first 20%, while going from a 20% reduction to a 40% reduction results in an additional decrease of the number of infections by 104 only. This huge difference in the effect of reducing the number of infections is caused by a phenomenon called *convexity*.



The number of infected people after 10 infection rounds is a convex function of the reproduction number R . This means that a first reduction of the reproduction rate R will always have a larger effect on the decrease of the number of infected people than a second reduction of the same size. This convexity property implies that moving from no masks to homemade masks saves more lives than going from homemade masks to more professional ones, provided that both steps decrease the reproduction number equally. More professional masks will definitely further decrease the number of infections, but most effect comes from the initial step. Hence, waiting to introduce the wear-a-mask policy until sufficient masks are available or until the highest quality masks are available is an unforgivable waste of time.

Let us now look at the situation where the reproduction number increases. This will happen in a lockdown exit strategy where some of the imposed restrictions are relaxed. Suppose that the reproduction number increases from 1.6 to 1.8. In this case, the number of infected people after 10 rounds of infections increases from 110 to 357, an increase of the number of infections by 247. The loss from loosening the effectiveness of the lockdown regulation is much larger than the gain from an improvement of the effectiveness of the policy by the same amount. Indeed, in case the reproduction number is decreased from 1.6 to 1.4, the number of infected people after 10 rounds decreases from 110 to 29, which is a decrease of the number of infections by 81. Summarizing, the extra number of infections after 10 rounds due to an increase of the reproduction number by 0.2 is more than 3 times higher than the decreased number of infections observed in case of a decrease of the reproduction number by 0.2. This is another illustration of the convexity property of the number of infections after a fixed number of rounds. A consequence of this convexity is that a lockdown exit strategy,

which will increase the reproduction number R , is a very delicate exercise that has to be performed very cautiously and step-by-step.

The calculations presented above clearly illustrate the importance of wearing masks as part of an efficient strategy to fight the virus. The model is definitely too simple to explain and take into account all effects of wearing masks. It cannot be applied for any number of infection rounds, as it does not take into account that after a sufficiently large number of steps, when a higher proportion of the population becomes immune, the effective reproduction number will automatically go down.

The numerical values we used for the decrease of the reproduction number due to introducing a wear-a-mask policy were chosen for illustrative purposes only. Important to notice however is that our conclusions about convexity remain to hold for any numerical value of this decrease, with different values leading to a faster or slower effect on fighting the virus.

While we focused on the wear-a-mask policy, it is clear that similar observations hold for any other strategy that influences the number of infected people. Any strategy changing the likelihood of one person infecting another has an exponential effect. A small decrease in the reproduction number may imply a huge decrease in deaths after a number of infection rounds.

Convexity implies that introducing a compulsory wear-a-mask policy in public spaces as soon as possible should be an essential part of an efficient lockdown exit strategy. This conclusion remains to hold even if these masks are homemade and even if we do not have statistical evidence about the magnitude of their effect. By not seizing this opportunity, we might miss the first plane ...

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