Implied co-movement behavior in stock markets[1]

The Herd Behavior Index

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[1] joint work with Daniel Linders, Wim Schoutens, ...

Main concepts and references

- **Comonotonicity**:
  - Dhaene, Denuit, Goovaerts, Kaas & Vyncke (2002a,b).
- **Super-replicating strategies for index options**:
  - Chen, Deelstra, Dhaene & Vanmaele (2008).
  - Linders, Dhaene, Hounnon, & Vanmaele (2012).
- **Measuring herd behavior in stock markets**:
  - Dhaene, Linders, Schoutens & Vyncke (2012).
  - Linders, Dhaene & Schoutens (2015).

Introduction

The millenium bridge (London, 2000)

- What is the probability that 1000 men walking at random on the bridge will end up walking in step?
- Soon after its opening, the bridge started to sway from side to side and had to be closed.

Introduction

Herd behavior in stock markets

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Herd behavior in stock markets

- Our goal: Define an index which:
  - reflects market's perception of degree of co-movement of stock prices,
  - is forward looking, based on observed option data,
  - is model-independent.
- We will call this index the **Herd Behavior Index** (HIX).
- The HIX may give information about the degree of diversification that is possible by investing in a portfolio of stocks.

The financial market

- The usual set up:

$$\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \leq t \leq T}, \mathbb{P}\right)$$

- Current time is denoted by 0. Time span is T years.
- The stock market:
  - n stocks.
  - $X_i(T) \equiv X_i =$ price of (dividend paying) stock i at time T.
- Assumptions:
  - The market is arbitrage-free.
  - The time - 0 price of any traded contingent claim with pay-off $A(T)$ at time T is given by

$$e^{-rT} \, \mathbb{E}\left[A(T)\right]$$

  - $r =$ risk-free interest rate (deterministic).
  - Expectation is taken w.r.t. the (unknown) measure $\mathbb{Q}$.

The financial market

Stock options

- European options on stock i:
  - Maturity T.
  - Arbitrage-free prices:

$$\boxed{C_i\,[K] = e^{-rT}\ \mathbb{E}[(X_i - K)_+]}$$

  and

$$\boxed{P_i\,[K] = e^{-rT}\ \mathbb{E}[(K - X_i)_+]}$$

- Traded strikes for stock i:

$$0 = K_{i,0} < K_{i,1} < \ldots < K_{i,m_i} < K_{i,m_i+1} = F_{X_i}^{-1}(1) < \infty$$

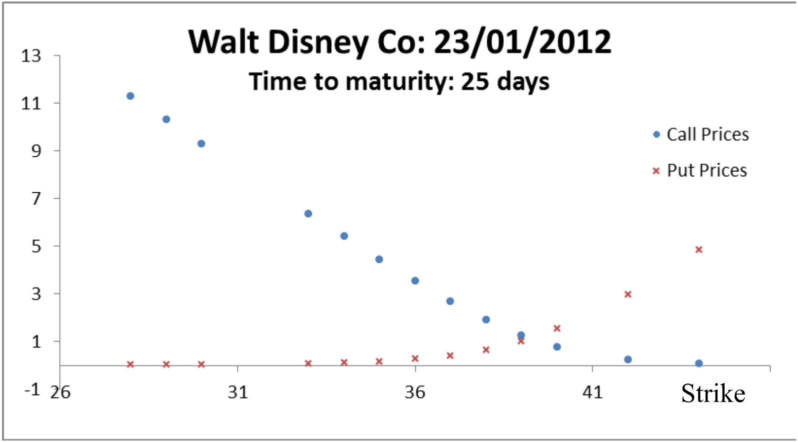
- Option curve C_i vs. option-implied distribution F_{X_i}:

$$\boxed{F_{X_i}(x) = 1 + e^{rT}\ C_i'[x+]}$$

The financial market

Stock options

- Options on Walt Disney:



The financial market

The stock market index and its options

- The stock market index:
  - $S(T) \equiv S =$ value of stock market index at time T:

$$\boxed{S = w_1 X_1 + \cdots + w_n X_n}$$

  - $w_i =$ positive weight factors.
- European options on the index:
  - Maturity T.
  - Arbitrage-free prices:

$$\boxed{C[K] = e^{-rT}\ \mathbb{E}[(S-K)_+]}$$

and

$$\boxed{P[K] = e^{-rT}\ \mathbb{E}[(K-S)_+]}$$

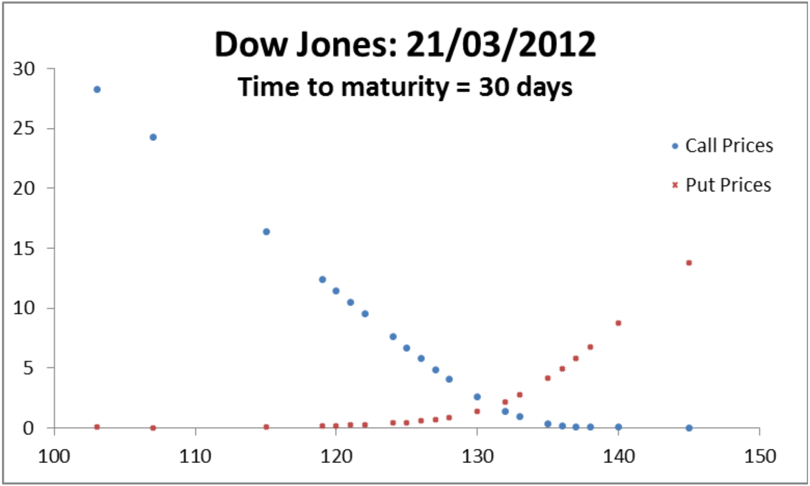
- Traded strikes:

$$K_{-l} < K_{-l+1} < \ldots < K_{-1} < K_0 \le \mathbb{E}[S] < K_1 < \ldots < K_{h-1} < K_h$$

The financial market

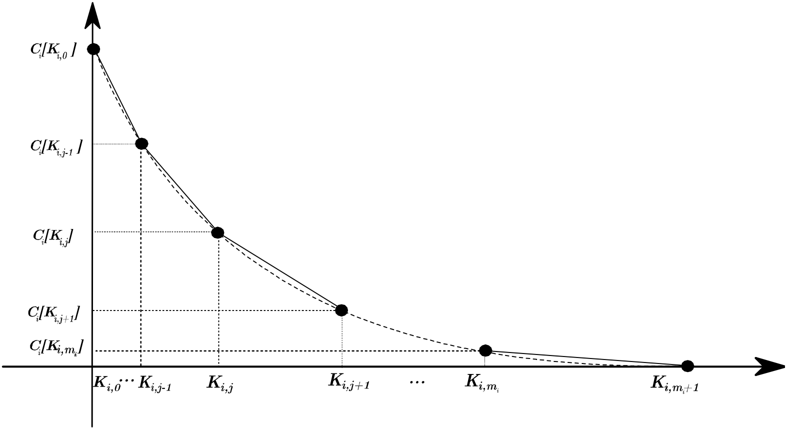
The stock market index and its options

- European options on the DJ index:



Option-implied stock price distributions

- We approximate the unknown option curve $C_i[K]$ of each stock i (dashed line) by the piecewise linear curve $\overline{C}_i[K]$ connecting the traded strikes (solid line).



Option-implied stock price distributions

- Approximation for $K \notin (0, K_{i,m_i+1})$:

$$\overline{C}_i[K] = C_i[K]$$

- '**Worst case' stock option curves**:
  - $\overline{C}_i[K]$ is convex and decreasing.
  - Any $\overline{C}_i[K]$ is a convex combination of observed option prices.
  - $\overline{C}_i[K]$ is the largest possible stock option price.
- '**Convex order' - largest risk neutral distributions**:
  - The option-implied cdf's $\overline{F}_{X_i}$ corresponding to the 'worst-case' option curves follows from:

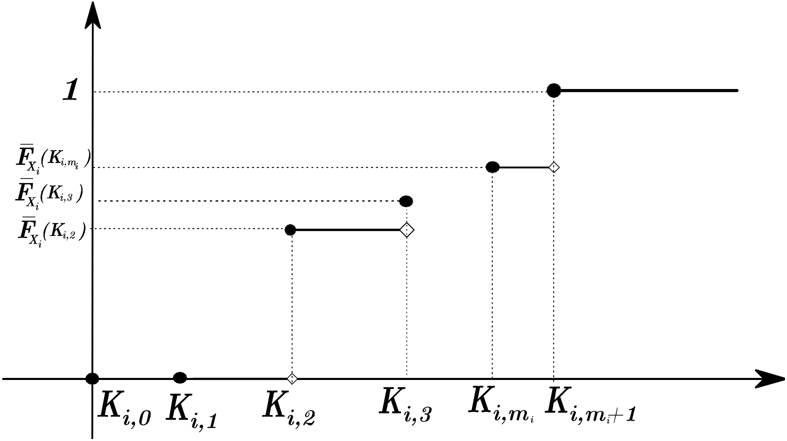
$$\boxed{\overline{F}_{X_i}(x) = 1 + e^{rT}\ \overline{C}_i'[x+]}$$

  - Convex order:

$$\boxed{F_{X_i} \leq_{cx} \overline{F}_{X_i}}$$

Option-implied stock price distributions

$$\boxed{\overline{F}_{X_i}(x) = 1 + e^{rT}\ \overline{C}_i'[x+]}$$



Forward contracts

- S = value of stock market index at time T.
- Consider the contingent claim with pay-off $f(S)$ at T.
- For any $a \geq 0$, this pay-off can be expressed as[2]:

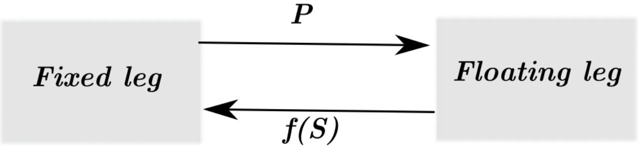
$$f(S) = f(a) + f'(a)\left[(S-a)_+ - (a-S)_+\right] + \int_0^a f''(K)(K-S)_+ \, \mathrm{d}K + \int_a^{+\infty} f''(K)(S-K)_+ \, \mathrm{d}K$$

- Model-free static replication of $f(S)$:
  - Buy $f(a)$ bonds, each paying an amount 1 at maturity.
  - Buy $f'(a)$ calls $C[a]$.
  - Sell $f'(a)$ puts $P[a]$.
  - Buy $f''(K)\,\mathrm{d}K$ puts $P[K]$, for $K < a$.
  - Buy $f''(K)\,\mathrm{d}K$ calls $C[K]$, for $K > a$.

[2] Carr & Madan (2001). Cheung, Dhaene, Kukush & Linders (2015).

Forward contracts

- Consider the swap contract, underwritten at time 0, with pay-offs at time T given by



- The time-0 price of the forward contract is 0:

$$P = \mathbb{E}\left[f\left(S\right)\right]$$

- The time-0 forward price in terms of option prices:

$$\begin{aligned}\mathbb{E}\left[f\left(S\right)\right] &= f\left(a\right) + e^{rT} f'\left(a\right)\left(C\left[a\right] - P\left[a\right]\right) \\ &\quad + e^{rT}\int_0^a f''\left(K\right) P\left[K\right] \mathrm{d}K \\ &\quad + e^{rT}\int_a^{+\infty} f''\left(K\right) C\left[K\right] \mathrm{d}K\end{aligned}$$

Forward contracts

- Traded strikes: $K_{-l}, K_{-l+1}, \ldots, K_{-1}, K_0, K_1, \ldots, K_{h-1}, K_h$.
- Approximation for $\mathbb{E}\left[f\left(S\right)\right]$: (composite trapezoidal rule)

$$\mathbb{E}\left[f\left(S\right)\right] \approx f\left(\mathbb{E}\left[S\right]\right) + e^{rT} \sum_{i=-l}^{h} f''\left(K_i\right) \Delta K_i Q\left[K_i\right] - \frac{f''\left(K_0\right)}{2}\left(\mathbb{E}[S] - K_0\right)^2$$

  - $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$ for $i = -l+1, \ldots, h-1$.
  - $\Delta K_{-l} = K_{-l+1} - K_{-l}$ and $\Delta K_h = K_h - K_{h-1}$.
  - $Q\left[K_i\right]$ defined by

$$Q\left[K_i\right] = \begin{cases} P\left[K_i\right], & \text{if } K_i < K_0 \\ \frac{C\left[K_i\right] + P\left[K_i\right]}{2} & \text{if } K_i = K_0 \\ C\left[K_i\right], & \text{if } K_i > K_0 \end{cases}$$

Forward contracts

An example

- The contract:
  - Floating leg:

$$\left(S - \mathbb{E}\left[S\right]\right)^2 = 2\left(\int_0^{\mathbb{E}[S]} \left(K - S\right)_+ \, dK + \int_{\mathbb{E}[S]}^{+\infty} \left(S - K\right)_+ \, dK\right)$$

  - Fixed leg:

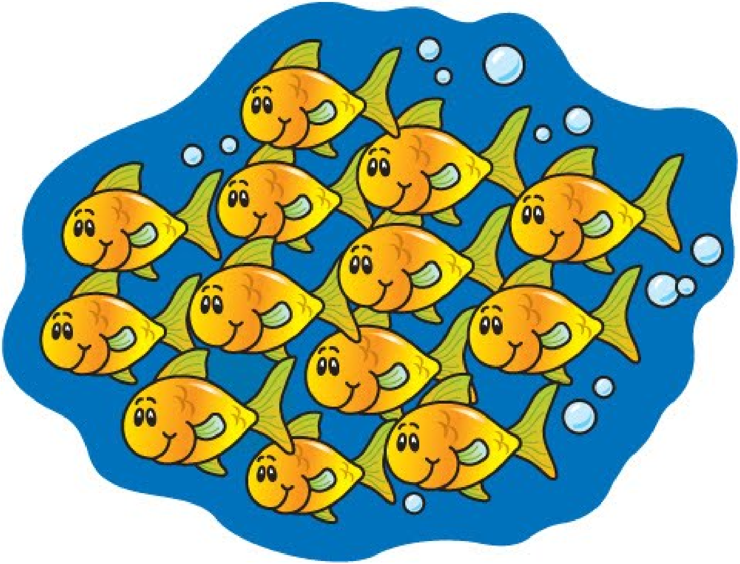
$$P = \mathrm{Var}[S] = 2e^{rT}\left(\int_0^{\mathbb{E}[S]} P\left[K\right] \, dK + \int_{\mathbb{E}[S]}^{+\infty} C\left[K\right] \, dK\right)$$

- Traded strikes: $K_{-l}, K_{-l+1}, \ldots, K_{-1}, K_0, K_1, \ldots, K_{h-1}, K_h$.
- Approximation:

$$\boxed{\mathrm{Var}[S] \approx 2e^{rT} \sum_{i=-l}^{h} \Delta K_i \; Q\left[K_i\right] - \left(\mathbb{E}[S] - K_0\right)^2}$$

  - ΔK_i and $Q\left[K_i\right]$ as defined above.

Comonotonicity



Comonotonicity

- Comonotonic stock prices[3]: (U is a uniform $(0,1)$ r.v.)

$$\boxed{(X_1, \ldots, X_n) \stackrel{\mathrm{d}}{=} \left(F_{X_1}^{-1}(U), \ldots, F_{X_n}^{-1}(U)\right)}$$

  - $\mathbb{P}$ - comonotonicity $\Leftrightarrow$ $\mathbb{Q}$ - comonotonicity.
  - Comonotonicity will not be used as an approximation, but as a point of reference.

- Comonotonicity gap[4]:
  - Distance between observed index option prices and comonotonic index option prices.
  - Problem: how to determine comonotonic index option prices?

[3] Dhaene, Denuit, Goovaerts, Kaas & Vyncke (2002a,b)

[4] Laurence (2008).

The comonotonic stock market index

- Consider $(X_1, \ldots, X_n)$ with risk neutral cdf's F_{X_i}.
- The stock market index:

$$S = \sum_{i=1}^{n} w_i X_i$$

- The comonotonic stock market index:

$$S^c = \sum_{i=1}^{n} w_i F_{X_i}^{-1}(U)$$

- **Theorem**[5]: The following statements are equivalent:
  - $(X_1, \ldots, X_n)$ is comonotonic.
  - $S \overset{\mathrm{d}}{=} S^c$.
  - $\mathrm{Var}[S] = \mathrm{Var}[S^c]$.

[5] Dhaene, Denuit, Goovaerts, Kaas, Vyncke (2002a); Cheung (2010).

The comonotonic stock market index

- The (approximated) comonotonic stock market index[6]:

$$\boxed{\overline{S}^c = \sum_{i=1}^n w_i \overline{F}_{X_i}^{-1}(U)}$$

- Algorithm[7]:

$$F_{\overline{S}^c}(K) = \max\left\{p \in A \mid \sum_{i=1}^n w_i \overline{F}_{X_i}^{-1}(p) \leq K\right\}$$

with

$$A = \left\{\overline{F}_{X_i}(K_{i,j}) \mid i = 1, \ldots, n \text{ and } j = 0, 1, \ldots, m_i\right\} \setminus \{0\}$$

[6] Hobson, Laurence & Wang (2005).
[7] Linders, Dhaene, Hounnon, Vanmaele (2012).

Comonotonic index option prices

- Definition:

$$\boxed{\overline{C}^c[K] = e^{-rT}\ \mathbb{E}[(\overline{S}^c - K)_+]}$$

- Decomposition formula[8]:

$$\boxed{\overline{C}^c[K] = \sum_{i=1}^n w_i \overline{C}_i[K_i^*]}$$

with $K_i^* = \overline{F}_{X_i}^{-1(\alpha_K)}\left(F_{\overline{S}^c}(K)\right)$ and α_K from $\sum_{i=1}^n w_i K_i^* = K$.

- Ordering relation:

$$\boxed{C[K] \leq \overline{C}^c[K]}$$

- Put options: similar.

[8] Dhaene, Wang, Young, Goovaerts (2000); Chen, Deelstra, Dhaene, Vanmaele (2008).

Comonotonic index option prices

$$\boxed{\overline{C}^c[K] = \sum_{i \in N_K} C_i\left[K_{i,j_i}\right] + \sum_{i \in \overline{N}_K}\left\{\alpha_K C_i\left[K_{i,j_i}\right] + (1-\alpha_K) C_i\left[K_{i,j_i+1}\right]\right\}}$$

- The integer j_i: the unique $j \in \{0, 1, \ldots, m_i + 1\}$ that satisfies

$$\overline{F}_{X_i}(K_{i,j-1}) < F_{\bar{S}^c}(K) \leq \overline{F}_{X_i}(K_{i,j})$$

- The set N_K:

$$N_K = \left\{ i \mid \overline{F}_{X_i}(K_{i,j_i-1}) < F_{\bar{S}^c}(K) < \overline{F}_{X_i}(K_{i,j_i}) \right\}$$

- The set $\overline{N}_K$:

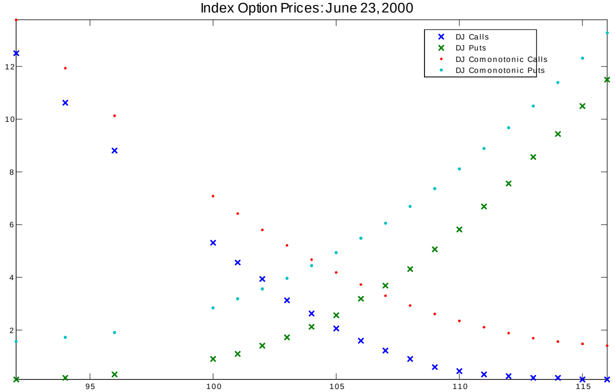
$$\overline{N}_K = \left\{ i \mid F_{\bar{S}^c}(K) = \overline{F}_{X_i}(K_{i,j_i}) \right\}$$

- The fraction α_K:

$$\alpha_K = 1 - \frac{K - \sum_{i=1}^n w_i K_{i,j_i}}{\sum_{i \in \overline{N}_K} w_i \left(K_{i,j_i+1} - K_{i,j_i}\right)}$$

Comonotonicity gap for DJ - index options

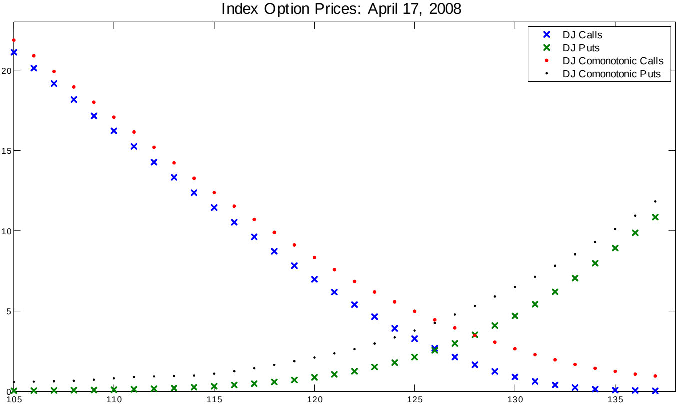
June 23, 2000



$$S(0) = 103.76, \;\; T = 30 \text{ days}, \;\; \text{HIX} = 0.286$$

Comonotonicity gap for DJ - index options

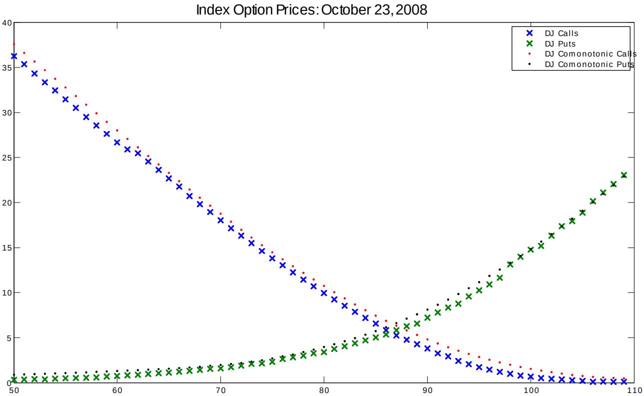
April 17, 2008



$$S(0) = 126, \;\; T = 30 \text{ days}, \;\; \text{HIX} = 0.369$$

Comonotonicity gap for DJ - index options

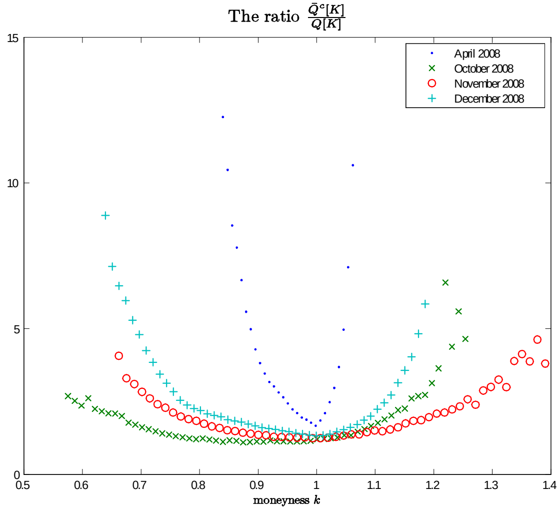
October 23, 2008



$$S(0) = 86.91, \; T = 30 \text{ days}, \; \mathrm{HIX} = 0.744.$$

Comonotonicity gap out-of-the-money DJ - index options

2008: April 17, October 23, November 20 and December 18.



gap (october) $<$ gap (november) $<$ gap (december) $<$ gap (april)

Comonotonic forward prices

- Expected value of functions of the comonotonic index price:

$$\mathbb{E}\left[f\left(\overline{S}^c\right)\right] = f\left(\mathbb{E}\left[S\right]\right) + e^{rT}\int_0^{\mathbb{E}[S]} f''\left(K\right)\overline{P}^c\left[K\right]dK + e^{rT}\int_{\mathbb{E}[S]}^{+\infty} f''\left(K\right)\overline{C}^c\left[K\right]dK$$

- Approximation:

$$\mathbb{E}\left[f\left(\overline{S}^c\right)\right] \approx f\left(\mathbb{E}\left[S\right]\right) + e^{rT}\sum_{i=-l}^{h} f''\left(K_i\right)\Delta K_i \overline{Q}^c\left[K_i\right] - \frac{f''\left(K_0\right)}{2}\left(\mathbb{E}[S] - K_0\right)^2$$

  - ΔK_i defined as before, while $\overline{Q}^c\left[K_i\right]$ defined by

$$\overline{Q}^c\left[K_i\right] = \begin{cases} \overline{P}^c\left[K_i\right], & \text{if } K_i < K_0, \\ \frac{\overline{C}^c[K_i]+\overline{P}^c[K_i]}{2} & \text{if } K_i = K_0, \\ \overline{C}^c\left[K_i\right], & \text{if } K_i > K_0. \end{cases}$$

Comonotonic forward prices

Example

- The function f:

$$f\left(\overline{S}^c\right) = \left(\overline{S}^c - \mathbb{E}[S]\right)^2$$

- Variance of the comonotonic index price at time T:

$$\mathrm{Var}[\overline{S}^c] = 2e^{rT}\left(\int_0^{\mathbb{E}[S]} \overline{P}^c[K]\, dK + \int_{\mathbb{E}[S]}^{+\infty} \overline{C}^c[K]\, dK\right)$$

- Approximation:

$$\mathrm{Var}\left[\overline{S}^c\right] \approx \left(\overline{s}^c\right)^2[T] \equiv 2e^{rT}\sum_{i=-l}^{h} \Delta K_i \overline{Q}^c[K_i] - \left(\mathbb{E}[S] - K_0\right)^2$$

  - ΔK_i and $\overline{Q}^c[K_i]$ as defined above.

The implied degree of herd behavior

- Measuring the degree of herd behavior: (f convex)

$$\boxed{\text{Degree of Herd Behavior} = \frac{\mathbb{E}\left[f\left(\frac{S}{\mathbb{E}[S]}\right)\right] - f(1)}{\mathbb{E}\left[f\left(\frac{S^c}{\mathbb{E}[S]}\right)\right] - f(1)}}$$

- Definition of the Herd Behavior Index[9]:

$$\boxed{\mathrm{HIX}_f\left[T\right] = \frac{e^{rT}\sum_{i=-l}^{h} f''\left(\frac{K_i}{\mathbb{E}[S]}\right)\Delta K_i \; Q[K_i] - \frac{f''\left(\frac{K_0}{\mathbb{E}[S]}\right)}{2}\left(\mathbb{E}[S]-K_0\right)^2}{e^{rT}\sum_{i=-l}^{h} f''\left(\frac{K_i}{\mathbb{E}[S]}\right)\Delta K_i \; \overline{Q}^c[K_i] - \frac{f''\left(\frac{K_0}{\mathbb{E}[S]}\right)}{2}\left(\mathbb{E}[S]-K_0\right)^2}}$$

  - Nominator:
    - Captures real market situation.
    - Follows from observed **index option prices**.
  - Denominator:
    - Captures comonotonic market situation.
    - Follows from observed **stock option prices**.

[9] Linders, Dhaene, Schoutens (2015).

The implied degree of herd behavior

Example

- Consider $f(s) = (s-1)^2$.
- Measuring the degree of herd behavior:

$$\boxed{\text{Degree of Herd Behavior} = \frac{\mathrm{Var}[S]}{\mathrm{Var}[S^c]}}$$

  - Properties:
    - This number takes values in $[0,1]$.
    - If $(w_1X_1, \cdots, w_nX_n)$ is a *joint mix*, it equals 0.
    - If $(w_1X_1, \cdots, w_nX_n)$ is *comonotonic*, it equals 1.
    - A higher value may indicate stronger positive dependency.
- Definition of the HIX:

$$\boxed{\mathrm{HIX}[T] = \frac{2e^{rT}\sum_{i=-l}^{h}\Delta K_i\ Q[K_i] - (\mathbb{E}[S]-K_0)^2}{2e^{rT}\sum_{i=-l}^{h}\Delta K_i\ \overline{Q}^c[K_i] - (\mathbb{E}[S]-K_0)^2}}$$

HIX vs. correlation

- Consider a market with the following risk-neutral dynamics:

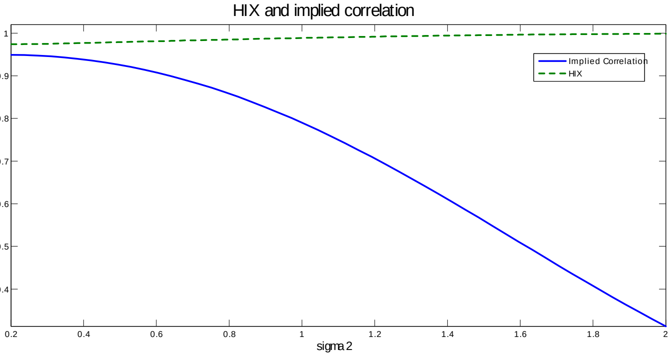
$$\begin{cases} \frac{\mathrm{d}X_1(t)}{X(t)} = 0.03\ \mathrm{d}t + 0.2\ \mathrm{d}B_1(t) \\ \frac{\mathrm{d}X_2(t)}{X_2(t)} = 0.03\ \mathrm{d}t + \sigma_2\ \mathrm{d}B_2(t) \end{cases}$$

  - $\{B_1(t), B_2(t)\}$: correlated Brownian motion process.
  - $\mathrm{covar}[B_1(t), B_2(t+s)] = 0.95t$ for all $t, s \geq 0$.
  - $T = 1$ year.
- Correlation between stock prices: $\mathrm{corr}[X_1, X_2]$.
- Herd Behavior Index:

$$\boxed{\mathrm{HIX}[1] = \frac{\mathrm{Var}[X_1+X_2]}{\mathrm{Var}\left[F_{X_1}^{-1}(U)+F_{X_2}^{-1}(U)\right]}}$$

HIX vs. correlation

$\text{Corr}[X_1, X_2]$ and $\text{HIX}[1]$ as a function of σ_2:



Implementation considerations

- Options prices are midquotes:

$$C[K] = \frac{C^{bid}[K] + C^{ask}[K]}{2}$$

- The T-year forward index price $\mathbb{E}[S]$:

$$\mathbb{E}[S] = e^{rT}\left(C[K^*] - P[K^*]\right) + K^*$$

  - with

$$K^* = \arg\min_{i \in \{-l,\dots,h\}} |C[K_i] - P[K_i]| \,.$$

- The T-year forward stock price $\mathbb{E}[X_i]$: similar.

Implementation considerations

- Suppose we want to calculate daily values of DJ - HIX$[T]$ for $T = 30$ calendar days.
- In general, options with maturity T are not available:
  - Consider near term T_1 and next term T_2 options.
  - HIX $[T]$ is a weighted average:

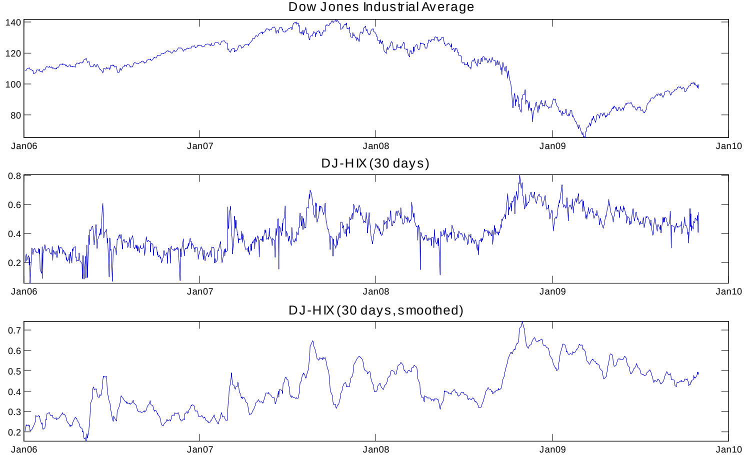
$$\text{HIX}[T] = \text{HIX}[T_1] \times \left[\frac{T_2 - T}{T_2 - T_1}\right] + \text{HIX}[T_2] \times \left[\frac{T - T_1}{T_2 - T_1}\right]$$

  - Roll to 2^{nd} and 3^{th} expiry dates if T_1 is less than 7 days.
- European index options and American stock options:
  - Replace (non-observed) $C_i\left[K_{i,j}\right]$ and $P_i\left[K_{i,j}\right]$ by the corresponding observed American option prices.

The DJ-HIX

Historical herd behavior

- Values of **DJ-HIX** for $T = \frac{30}{365}$.
- <u>Time period</u>: January, 2006 - October, 2009.



DJ-HIX vs. DJ-VIX

- **Realized Variance**: $\left(T = \frac{30}{365}\right)$

$$\mathrm{RV}\left[T\right] = \frac{1}{T}\sum_{j=1}^{365\times T}\left[\ln S\left(\frac{j}{365}\right) - \ln S\left(\frac{j-1}{365}\right)\right]^2$$

- **VIX**[10] (the Fear Index):

$$\begin{aligned} \mathrm{VIX} &= 100 \times \sqrt{\text{approximation of } \mathbb{E}\left[\mathrm{RV}\left[T\right]\right]} \\ &= 100 \times \sqrt{\frac{2}{T}e^{rT}\sum_{i=-l}^{h}\frac{\Delta K_i}{K_i^2}Q\left[K_i\right] - \frac{1}{T}\left(\frac{\mathbb{E}\left[S\right]}{K_0} - 1\right)^2} \end{aligned}$$

  - VIX is a barometer for implied volatility of the **S&P 500**, based on options on this index.
  - The VIX methodology can be applied to any index or stock with appropriate underlying options.

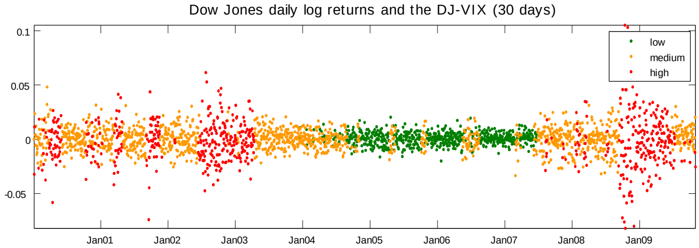
[10] CBOE (2009), Carr & Wu (2006)

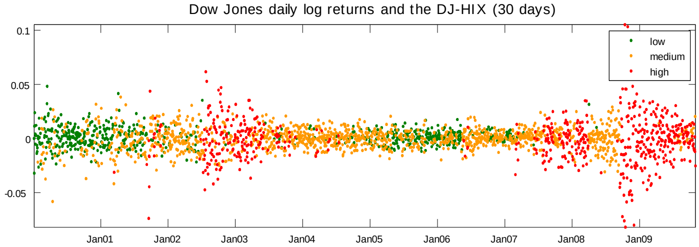
DJ-HIX vs. DJ-VIX

- DJ-VIX $=$ VIX-based volatility barometer for DJ.
- We always take $T = \frac{30}{365}$.

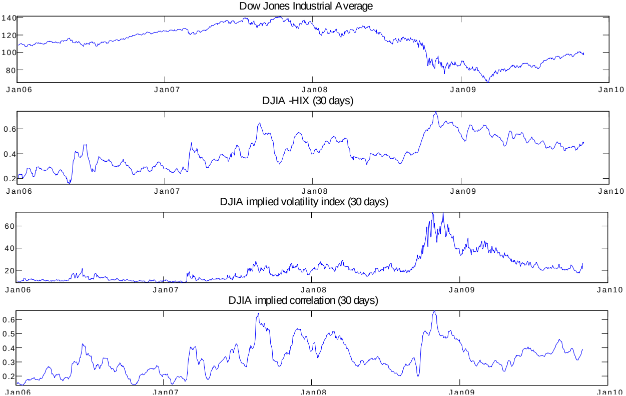
- <u>On next slide</u>: **Daily DJ - logreturns** over the period January, 2000 - October, 2009.
  - <u>Figure 1</u>: color of logreturns indicate level of **<u>VIX</u>**:
    - *Green*: historically low: VIX $\leq$ 0.1 quantile.
    - *Orange*: intermediate.
    - *Red*: historically large: VIX $\geq$ 0.9 - quantile.
  - <u>Figure 2</u>: color of logreturns indicate level of **<u>HIX</u>**:
    - *Green*: historically low: HIX $\leq$ 0.1 quantile.
    - *Orange*: intermediate.
    - *Red*: historically large: HIX $\geq$ 0.9 - quantile.

DJ-HIX vs. DJ-VIX





DJ-HIX vs. DJ-VIX vs. DJ-Implied Correlation



Conclusions

- **HIX**:
  - reflects market's perception of future co-movement behavior of stock prices,
  - is forward looking, based on observed option data,
  - HIX is model-independent.
- HIX vs. other implied measures:
  - HIX is a better implied measure for co-movement than implied correlation.
  - HIX and VIX give complementary information.
- Applications:
  - Using HIX as a barometer.
  - Using HIX as a buy/sell signal for the index.
  - Trading the comonotonicity gap.

Thank you for your attention

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