Moral hazard in peer-to-peer insurance with social connection

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Abstract

By gathering communities of acquainted participants, peer-to-peer (P2P) insurance has been providing partially refundable insurance coverage associated with the social networks among its participants. This study analyzes the issue of moral hazard within the framework of peer-to-peer insurance from a theoretical perspective. We investigate how the social network within the community affects the participants' incentive to spend effort on precautionary loss prevention. Using a quantitative framework to study the moral hazard in P2P insurance, we present that the participants' efforts at Nash equilibria are influenced by their social network. In addition, we investigate how participants' tendency to spend effort in reducing risk varies according to the structure of the social network.

Keywords: Moral hazard; Social network; Peer-to-peer insurance; InsurTech.
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1 Introduction

The emergence of InsurTech has brought forth innovative business models equipped with advanced technologies, aiming to disrupt the traditional insurance industry. Among these models, peer-to-peer (P2P) insurance stands out for its potential to alleviate moral hazard by gathering a network of members and reducing the role of conventional insurers. P2P insurance represents a novel risk-sharing network where individuals with similar risk profiles pool their resources to insure against common risks. The National Association of Insurance Commissioners (NAIC) defines P2P insurance as a product that enables a group of insured individuals to pool resources together and collaboratively manage their insurance arrangements (National Association of Insurance Commissioners 2023). This innovative InsurTech model facilitates risk-sharing among like-minded participants, introducing elements of control, trust, and transparency while reducing costs (Fang, Qin, Wu & Yu 2020).

One notable feature of P2P insurance is the formation of communities comprising acquainted participants. P2P insurance pools usually consist of a limited number of individuals, ranging from dozens to hundreds. Within these pools, diverse social networks emerge, often through invitations by friends or relatives. Consequently, the extent of social connections among participants differs, as some individuals are acquainted with more pool members than others. This self-organizing structure is believed to mitigate moral hazard, a persistent issue in the insurance industry.

This paper investigates the relationship between social connection and moral hazard in P2P insurance. Moral hazard, a long-standing problem in the insurance industry, incentivizes risk-taking behaviour among insured individuals. It arises when having insurance encourages the insureds to reduce their efforts to avoid the occurrences of losses, thereby raising the likelihood of accidents happening. Economists contend that resolving information asymmetry, which prevents insurers from perfectly observing insureds' efforts, is key to mitigating this problem (Arrow 1963, Pauly 1968, Research and Markets 2018, Thakor 2020, International Association of Insurance Supervisors 2017, Moenninghoff & Wieandt 2013, Institute of International Finance 2015, World Bank Group 2018). It is anticipated that this effort reduction (a.k.a. moral hazard) might be less pronounced if losses are partly shared among individuals known to each other, rather than fully burdened by a traditional insurer. Therefore, our objective is to analyze whether the social network within the risk-sharing community could enable P2P insurance to better address the issue of moral hazard.

This study conducts a theoretical analysis of the moral hazard problem in P2P insurance. We begin by formulating an optimal decision-making problem regarding participants' efforts in P2P insurance within specific social network structures. Subsequently, we offer a comprehensive examination of the Nash equilibria concerning participants' efforts. Our analysis reveals that the inclinations of P2P insurance participants to exert efforts are closely linked to their social connections within the network. Specifically, participants with larger social connections or greater centrality are more inclined to expend effort in minimizing losses. Additionally, each participant exhibits increased effort when the connectivity between any two participants strengthens. Thus, our theoretical analysis underscores that the social network significantly contributes to the low-cost advantage of the P2P insurance model. Furthermore, we investigate how participants' propensity to expend effort in risk reduction is influenced by the size of the P2P insurance pool. We discover that participants' efforts to mitigate loss probabilities diminish as the number of participants approaches infinity. In summary, our study enhances the understanding of how P2P insurance addresses moral hazard. We ascertain that the social network within the P2P insurance pool mitigates the moral hazard issue by enabling acquainted participants to collaborate and pool their risks.[1]

Our research primarily aligns with two main distinct lines of literature. Firstly, this paper joins the discussion on ex-ante moral hazard in insurance markets. In this scenario, an insurer compensates an insured based on the loss influenced by the insured's unobservable effort exerted before the loss happens, thereby providing the insured with

[1]This aligns with industry practices, as Friendsurance, a P2P insurance company, processes 20% to 40% fewer claims than others, partly because friends are less likely to harm or cheat each other (source: see https://www.fastcompany.com/3021024/a-social-network-for-insurance-that-cuts-costs-and-reduces-fraud).

an incentive to put less effort into preventing loss (Arrow 1963, Pauly 1968, Bolton & Dewatripont 2005). Since supervision is often costly, insurers commonly employ partial coverage to mitigate this effort-reduction behaviour (a.k.a. moral hazard). Ehrlich & Becker (1972) demonstrated that the insurance coverage reduces the insured's effort level if she is not prohibitively risk-averse; Helpman & Laffont (1975), Shavell (1979), and Arnott & Stiglitz (1988) revealed that the optimal market equilibria, in the presence of moral hazard, typically involve partial insurance coverage. In addition to theoretical research, numerous empirical studies have contributed to identifying moral hazard in the field of insurance practice (see e.g. Cummins & Tennyson (1996), Abbring, Heckman, Chiappori & Pinquet (2003), Abbring, Chiappori & Pinquet (2003), Kim, Kim, Im & Hardin (2009) and Spindler, Winter & Hagmayer (2014)).

In particular, our work is closely aligned with the literature that examines moral hazard in mutual insurance frameworks. Unlike traditional insurance arrangements, participants in mutual insurance collectively share risks and benefits as residual claimants, in which moral hazard is believed to be more effectively deterred. Cabrales, Calvó-Armengol & Jackson (2003) analyzed fire mutual insurance in Andorra and found that Nash equilibria often deviate from Pareto efficient outcomes; Lee & Ligon (2001) explored the relationship between the number of participants and their effort levels in mutual insurance; von Bieberstein, Feess, Fernando, Kerzenmacher & Schiller (2019), building upon Lee & Ligon (2001), further studied the scenario where participants in mutual insurance can freely choose the level of coverage.

Secondly, this paper contributes to the growing volume of literature on risk-sharing mechanisms in P2P insurance. P2P insurance represents a hybrid system that integrates both risk-sharing and risk-transfer mechanisms (Denuit 2019, Feng et al. 2022, Feng 2023). Recent studies have explored the optimal fusion of risk sharing and risk transfer rules in P2P insurance, see e.g. Denuit & Dhaene (2012), Denuit (2019), Denuit & Robert (2021*a*), Denuit et al. (2022), Denuit & Robert (2021*c*) and Chen, Feng, Hu & Mao (2023). While risk sharing has been a longstanding focus in risk management literature, it has gained renewed attention in the context of decentralization trends. Notably, Denuit et al. (2022) and Feng et al. (2022) provided comprehensive analyses of various risk-sharing rules and their properties, including the conditional mean risk-sharing rule proposed by Denuit & Dhaene (2012), the quantile-based risk sharing rule introduced by Denuit, Dhaene & Robert (2022). [2] Moreover, Abdikerimova & Feng (2022*a*) investigate the actuarially fair risk sharing rule for heterogeneous risks across multiple groups. Abdikerimova, Boonen & Feng (2024) extend the risk sharing rule to a multi-period framework. At last, researchers have also applied these theoretical frameworks to practical insurance scenarios, such as property & casualty insurance (Denuit & Robert 2021*b*) and flood risk pooling (Feng, Liu & Taylor 2023).

The remainder of this paper proceeds as follows. Section 2 introduces the mechanism of P2P insurance. In Section 3, we develop a theoretical framework to analyze the moral hazard problem in P2P insurance. Section 4 and 5 are dedicated to equilibria analysis and comparative statics. Section 6 presents a numerical illustration. Finally, Section 7 concludes with a summary.

2 Framework of P2P insurance with social network

P2P insurance is a concept that was likely first proposed in 2010 by *Friendsurance*, a German InsurTech start-up. Since then, it has subsequently expanded to many other countries. Since the introduction of P2P insurance, numerous InsurTech companies around the world have followed suit. It allows a small group of family members or friends with the same needs of insurance to collectively pool their risks in a P2P platform with a (re-) insurance contract, see e.g. Denuit (2019), Denuit & Robert (2021*b*), Denuit, Dhaene & Robert (2022), Denuit, Dhaene, Ghossoub & Robert (2023).

2.1 Model setting

There are n individuals, also referred to as participants or members, numbered from 1 to n, who share a common concern for financial losses. The set of participants (or members)

[2] Additionally, Jiao, Kou, Liu & Wang (2022) axiomatized the conditional mean risk-sharing rule, while Dhaene, Robert, Cheung & Denuit (2023) axiomatized the quantile-based risk sharing rule.

is denoted by $\mathcal{M}_n = \{1, 2, \ldots, n\}$. Participants face two outcome states at the end of a single period, i.e. "loss" and "no loss". We use a binary variable I_i to indicate participant i's outcome state at the end of the period, where $I_i = 1$ if the participant's outcome state is "loss", and $I_i = 0$ in case of "no loss". All participants experience the same level of loss severity, meaning they each incur a deterministic loss amount $\ell \in \mathbb{R}_{++}$ when a loss happens. We consider that participants collectively pool their risks into a P2P platform. The aggregated risk pooled by all participants, denoted by S, is calculated using the expression

$$S = \ell \sum_{i=1}^{n} I_i$$

Notably, the aggregated risk S (or aggregated loss after realization) is a random variable to be observed until its realization.

2.2 Mechanism of P2P insurance

We mathematically introduce the mechanism of P2P insurance. P2P insurance is a hybrid system that integrates risk sharing and risk transfer. In a P2P insurance pool, participants assemble their risks into an aggregated risk, which is partly transferred to an insurer, with the remainder shared among the participants. Figure 1 presents the mechanism of P2P insurance. The nodes represent participants, the Ⓟ refers to the P2P platform, and the Ⓘ indicates the insurer. The straight arrows refer to the momentary transfer between entities, while the curved arrows symbolize the social connections between participants, which will be introduced in the next section.

P2P insurance applies the excess-of-loss risk transfer rule to cap the aggregated risk (or loss) S. This means that the platform transfers the aggregated risk S beyond a deterministic threshold $D \in \big[0, \sup(S)\big] = [0, nl]$ to an insurer by paying a deterministic aggregated premium $\Pi \in \mathbb{R}_+$. Following the transfer of the tail risk $(S-D)_+$ at a premium cost Π, the pool of participants only needs to be responsible for the residual risk:

$$S - (S - D)_+ = \min(S, D)$$

This residual risk is to be shared among participants. Given that the aggregated loss can potentially be very large if left uncapped, deterring participants from joining the pool, the excess-of-loss risk transfer rule is considered necessary to cap it.

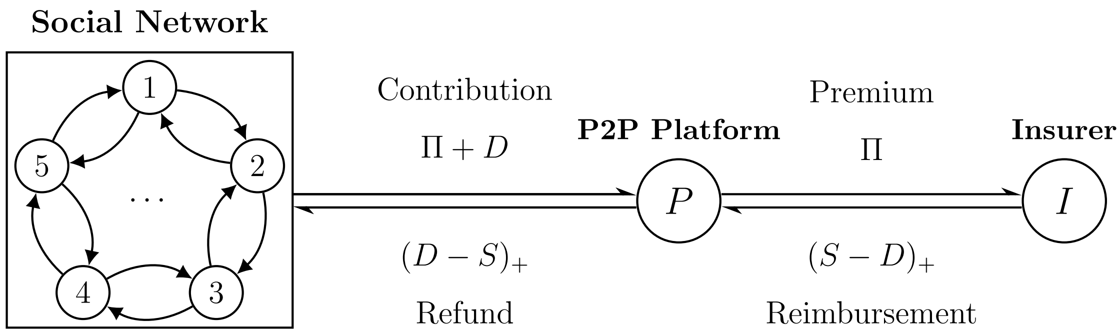

Figure 1: Mechanism of P2P insurance

Considering the homogeneity of loss among participants, P2P insurance adopts the uniform risk-sharing rule to allocate the residual risk. To ensure the enforceability of this risk-sharing mechanism, the P2P platform generally requires each of the n participants to initially contribute $\frac{\Pi+D}{n}$, constituting the aggregated contribution $(\Pi + D)$. This aggregated contribution is firstly used to pay the aggregated premium Π. The remaining amount, called an aggregated deposit and denoted as D, is then used to cover the residual loss, expressed as $\min(S, D)$. Consequently, under this arrangement, any aggregated loss beyond D, denoted as $(S - D)_+$, is reimbursed by the insurer, while any aggregated loss below D, denoted as $\min(S, D)$, is shared among all participants.

At the end of the period, participants may expect a refund from the P2P platform if there is a surplus in the aggregated deposit, i.e. $S < D$. In this case, the leftover aggregated deposit, denoted as $(D - S)_+$, is refunded and uniformly shared among all participants, with no liability incurred by the insurer. However, if the aggregated loss exceeds the aggregated deposit, i.e. $S \geq D$, then no refund is issued and the insurer is obligated to reimburse the excess aggregated loss, expressed as $(S - D)_+$, to the platform.

Therefore, we can summarize each of the n participant's costs for P2P insurance coverage. Taking into account the possible refund, each participant actually pays

$$\frac{\Pi + D}{n} - \frac{(D - S)_+}{n} = \pi + \min\big(r^{\mathrm{uni}}(S), d\big)$$

Here, the individual deposit is represented as $d := \frac{D}{n}$, the individual premium as $\pi := \frac{\Pi}{n}$, and the individual shared loss as $r^{\text{uni}}(S) := \frac{S}{n}$. In essence, the cost for P2P insurance coverage, represented as $\pi + \min\big(r^{\text{uni}}(S), d\big)$, is a random variable contingent upon the realization of S. Let the initial wealth of participants be w, then, each participant's final wealth, denoted as w_f, is given by

$$w_f = w - \pi - \min\big(r^{\text{uni}}(S), d\big)$$

It is worth noting that each participant is fully insured after paying $\pi + \min\big(r^{\text{uni}}(S), d\big)$.

2.3 Social network

The so-called P2P insurance model is based on the concept that participants in a social network, typically groups of family members or friends, pool their resources to compensate each other for losses and cut down the cost of insurance (Abdikerimova & Feng 2022*b*). Many P2P insurance platforms encourage friends and family members to join mutual groups (Biener, Eling, Landmann & Pradhan 2018) Thus, some specific forms of social networks exist in the P2P insurance model as participants are often friends or relatives.[3] Though a certain form of social networks exists in P2P insurance, it does not mean that everyone directly knows each other in the pool. By geometric characterization, each participant can be viewed as a node in the pool and any two are connected if they know each other (see Figure 1).

We use a graph to mathematically model the social network that exists in a P2P pool consisting of n participants. For any two distinct participants, say i and j, $g_{ij} \in \mathbb{R}_+$ denotes the constant degree of i's social connection toward j, indicating how much i cares about j. The collection of their cross-connections forms a graph represented by an $n \times n$

[3]This assumption aligns with industry practices, as seen with Friendsurance, a P2P insurance company that connects friends via social media to purchase collective non-life policies from established insurers. By joining a group they know and trust, the likelihood of dishonest behaviour is reduced (source: see `https://www.the-digital-insurer.com/dia/friendsurance-germany-makes-insurance-social-again/`.)

matrix $\mathbf{G}$[4] where

$$\mathbf{G} := \begin{bmatrix} 0 & g_{12} & \cdots & g_{1n} \\ g_{21} & 0 & \cdots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & 0 \end{bmatrix} \tag{1}$$

Notably, we employ a directed and weighted graph $\mathbf{G}$ to represent the social network underlying P2P insurance.[5] The centrality of participant $i \in \mathcal{M}_n$ is expressed as her aggregated social connections toward others, denoted as $\sum_j g_{ij}$. Thus, we say a participant has a greater centrality in the social network if she possesses stronger social connections with others.

3 Moral hazard problem formulation

The problem of moral hazard has gained substantial scholarly attention within the existing literature, particularly in the domain of insurance contracts. This issue holds particular relevance in the context of P2P insurance arrangements. In this section, our objective is to investigate the decision-making processes that govern how efforts are allocated in the context of P2P insurance.

In P2P insurance, efforts to reduce losses have the characteristics of public goods. This is because when each participant exerts effort to reduce her own risk, it creates positive externalities by lowering the aggregated risk for the entire risk pool. Remarkably, these externalities may be endogenized by the altruistic behaviour of the participants in the P2P insurance.

[4] We set $g_{ii} = 0$ for all $i \in \mathcal{M}_n$ because each participant needn't redundantly connect to herself again, see equation (2) below.

[5] We explicitly consider the social network as given, a setting widely adopted in the literature, such as by Bramoullé, Kranton & D'Amours (2014) and Bramoullé & Kranton (2007).

3.1 Effort spending in P2P insurance

Each participant can spend a continuous and unobservable effort $x \in \mathbb{R}_+$ at a cost to reduce her loss probability. The cost of effort, denoted as $c(x)$, is twice continuously differentiable, strictly increasing and convex regarding x, with $c(0) = 0, c'(x) > 0$ and $c''(x) > 0$. The probability of loss, denoted as $q(x) \in (0, 1)$, is also twice continuously differentiable and decreases with the effort in a diminishing way, i.e., $q'(x) < 0$ and $q''(x) > 0$. For simplicity, we let $\boldsymbol{x} := (x_1, x_2, \ldots, x_n) \in \mathbb{R}_+^n$ be the effort levels of all participants, that can be further abbreviated as $\boldsymbol{x} = (x_i, \boldsymbol{x}_{-i})$ with $\boldsymbol{x}_{-i} := (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) \in \mathbb{R}_+^{n-1}$.[6]

With the effort to reduce the probability of loss, the loss indicator of a representative participant i is denoted as

$$I_i \sim \text{Bernoulli}\big(q(x_i)\big),$$

for any $i \in \mathcal{M}_n$. Let $\boldsymbol{q}(\boldsymbol{x}) = \big(q(x_1), q(x_2), \ldots, q(x_n)\big) \in (0, 1)^n$, then, the aggregated loss incurred by n participants, denoted as $S = \ell \sum_{i=1}^n I_i$, can be described as a deterministic loss amount, i.e. ℓ, multiplied by the sum of individual losses

$$\sum_{i=1}^{n} I_i \sim \text{PBD}\big(\boldsymbol{q}(\boldsymbol{x})\big),$$

which follows a Poisson Binomial distribution.[7] Here, $I_1, \ldots, I_n$ are mutually independent but not necessarily identically distributed due to the potentially heterogeneous efforts exerted by participants. Using this characterization, we know that the distribution of the shared loss, i.e. $r^{\text{uni}}(S) = \frac{S}{n}$, is also dependent on the effort levels $\boldsymbol{x}$ of all participants.

3.2 Altruistic utility with social network

Considering a certain social network exists in the pool consisting of n participants, we assume that participants are altruistic, that is, they make effort decisions by maximizing

[6] Hereafter in our study, $\boldsymbol{x} \geq \boldsymbol{y}$ means $x_i \geq y_i$ for all i when comparing two vectors $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ and $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)$.

[7] $\text{PBD}(\boldsymbol{q}(\boldsymbol{x}))$ denotes the Poisson Binomial distribution, representing the discrete probability distribution of a sum of independent Bernoulli trials that are not necessarily identically distributed, i.e. $I_i \sim q(x_i)$.

not only their own utilities but also their peers' utilities(Bergstrom 1989, Hori 2001, Simon 2016). Let $v_i(\boldsymbol{x})$ be participant i's personal utility function, then her altruistic utility function is expressed as:

$$u_i(x_i, \boldsymbol{x}_{-i}; \mathbf{G}) := v_i(\boldsymbol{x}) + \sum_j g_{ij} v_j(\boldsymbol{x}), \text{ for all } i \in \mathcal{M}_n \tag{2}$$

Because each participant contributes the cost for coverage, denoted as $\pi + \min\big(r^{\text{uni}}(S), d\big)$, for the full coverage, and pays the cost of effort $c(x_i)$ to reduce her probability of loss, the personal utility function of a representative participant i is represented by: [8]

$$\begin{aligned} v_i(\boldsymbol{x}) &:= \mathrm{E}\big[b(w_f) \mid \boldsymbol{x}\big] - c(x_i) \\ &= \mathrm{E}\Big[b\big(w - \pi - \min(r^{\text{uni}}(S), d)\big) \mid \boldsymbol{x}\Big] - c(x_i) \end{aligned}$$

for all $i \in \mathcal{M}_n$.[9] $b(\cdot)$ is assumed to be a twice continuously differentiable, strictly increasing ($b'(\cdot) > 0$), and strictly concave ($b''(\cdot) < 0$) Von Neumann–Morgenstern utility function. Each participant, say $i \in \mathcal{M}_n$, non-cooperatively selects her optimal effort x_i^* to maximize her altruistic utility function, given others' effort levels $\boldsymbol{x}_{-i}$:

$$x_i^* = \arg\max_{x_i \in \mathbb{R}_+} u_i(x_i, \boldsymbol{x}_{-i}; \mathbf{G}), \text{ for all } i \in \mathcal{M}_n$$

Because efforts exhibit characteristics of public goods, one may think that any participant's high effort to reduce the aggregated loss can discourage others from doing the same thing. This assertion is commonly acknowledged in the realm of the public good provision, see for example Bramoullé, Kranton & D'Amours (2014). However, in this paper, we will demonstrate that one's high effort could encourage others to exert more effort.

It is believed that P2P insurance probably mitigates the conflicts that exist between traditional insurers and insureds. In traditional insurance, the incentives of the insurer

[8] Because the cost of effort is reasonably unrelated to the risk appetite, participant i's personal utility is assumed to be additively separable in money and effort(Bourgeon & Picard 2014).

[9] As the effort levels $\boldsymbol{x}$ paid by participants have an impact on the distribution of shared loss $r^{\text{uni}}(S)$, we deliberately use "$\mid \boldsymbol{x}$" to emphasize that the expected utility level of the final wealth, i.e. $\mathrm{E}\big[b(w_f) \mid \boldsymbol{x}\big]$, is dependent of $\boldsymbol{x}$, although $\boldsymbol{x}$ is *not* a random vector. This notation is also used by (Bolton & Dewatripont 2005, page 142).

and the insured may not always align. This is because the insured pays a fixed premium to the insurer regardless of whether a loss occurs, and the insured does not share the insurer's profit if she incurs no or few loss outcomes. However, in P2P insurance, this issue is effectively addressed. Each participant pays a lower cost for coverage if fewer claims are filed. This incentivizes altruistic participants to exert more effort in reducing the aggregated loss shared by their relatives or friends, thus reducing their burden. Thus, it is expected that the social networks underlying P2P insurance pools can mitigate moral hazard and consequently lower the aggregated loss (Moenninghoff & Wieandt 2013).

3.3 Trade-offs in effort choices

We delve into the trade-off faced by participants in P2P insurance when deciding how much effort they should exert. In navigating this trade-off, participants must strategically optimize their altruistic utility functions. Opting for higher effort entails increased reciprocal altruistic and pure altruistic benefits but also incurs a higher cost of effort.

Firstly, committing to higher effort levels to mitigate losses introduces an additional price of the cost of effort. Since $c'(x) > 0$, a participant opting for greater effort incurs the higher initial cost of effort, which is represented by $c(x)$.

Secondly, when a typical participant, say $i \in \mathcal{M}_n$, chooses to exert higher effort, it results in a reduced shared loss, denoted as $r^{\text{uni}}(S)$, for herself, thereby leading to a reciprocal altruistic advantage. As previously established, participants' shared loss is expressed as $r^{\text{uni}}(S) = \frac{\ell(I_i+\sum_{j\neq i} I_j)}{n}$, where $I_i \sim \text{Bernoulli}\big(q(x_i)\big)$. Since $q'(x) < 0$, it indicates that participant i who exerts greater effort, represented by x_i, is expected to bear a lower shared loss. Thus, it can be inferred that by increasing effort, a participant could enhance her personal utility by shouldering a reduced shared loss in an expectational sense.

Thirdly, opting for high effort actively contributes to the reduction of the shared loss for other participants, aligning with pure altruistic preferences. Because each participant's shared loss is influenced by the efforts paid by other participants, the higher effort exerted by a representative participant, say $i \in \mathcal{M}_n$, can decrease the shared loss borne by others.

Consequently, participant i benefits from her pure altruistic behaviour, reaping rewards beyond personal gains.

4 Nash equilibrium analysis

4.1 Problem formulation

We present a formal characterization of Nash equilibrium concerning participants' efforts (or "effort equilibrium" for short). Mathematically, a Nash equilibrium point for participants' efforts, denoted by $\boldsymbol{x}^* \in \mathbb{R}_+^n$, is defined by the following condition:

$$u_i(\boldsymbol{x}^*; \mathbf{G}) = \max_{x_i \in \mathbb{R}_+} \left\{ u_i(x_i, \boldsymbol{x}_{-i}^*; \mathbf{G}) \mid (x_i, \boldsymbol{x}_{-i}^*) \in \mathbb{R}_+^n \right\}, \quad \text{for all } i \in \mathcal{M}_n \tag{3}$$

The condition (3) signifies that at a Nash equilibrium point, denoted as $\boldsymbol{x}^*$, no participant can unilaterally increase her altruistic utility by altering her effort level. It is noteworthy that due to the strict concavity of the altruistic utility function with respect to x_i, mixed strategy Nash equilibria are precluded. Consequently, our analysis concentrates solely on the investigation of pure-strategy Nash equilibria. Thus, a Nash equilibrium is attained when a vector $\boldsymbol{x}^* = (x_1^*, \ldots, x_n^*)$ exists, adhering to the condition stipulated in the condition (3).

To find the Nash equilibria, we introduce the notation:

$$B_{-i} := \sum_{j \neq i} I_j \sim \text{PBD}\Big(\boldsymbol{q}_{-i}\big(\boldsymbol{x}_{-i}\big)\Big)$$

where $\boldsymbol{q}_{-i}\big(\boldsymbol{x}_{-i}\big) = \big(q(x_1), \ldots, q(x_{i-1}), q(x_{i+1}), \ldots, q(x_n)\big) \in (0,1)^{n-1}$. Leveraging the iterated expectation theorem, we can reformulate the altruistic utility function as follows:

$$\begin{aligned} u_i(x_i, \boldsymbol{x}_{-i}; \mathbf{G}) &= \mathrm{E}\Big[b\Big(w - \pi - \min\big(r^{\text{uni}}(S), d\big)\Big) \mid \boldsymbol{x}\Big] - c(x_i) \\ &\quad + \sum_j g_{ij} \Big\{ \mathrm{E}\Big[b\Big(w - \pi - \min\big(r^{\text{uni}}(S), d\big)\Big) \mid \boldsymbol{x}\Big] - c(x_j) \Big\} \\ &= (1 + \sum_j g_{ij}) \Big\{ q(x_i) \mathrm{E}\Big[b\Big(w - \pi - \min\big(\frac{\ell + \ell B_{-i}}{n}, d\big)\Big) \mid \boldsymbol{x}_{-i}\Big] \end{aligned}$$

$$+ \big(1-q(x_i)\big)\mathrm{E}\Big[b\Big(w-\pi-\min\big(\frac{\ell B_{-i}}{n},d\big)\Big) \mid \boldsymbol{x}_{-i}\Big]\Big\} - c(x_i) - \sum_j g_{ij}c(x_j)$$

For ease of exposition, let $\delta(x_i, \boldsymbol{x}_{-i})$ represent the marginal reciprocal altruistic benefit of participant i. It is expressed as:

$$\delta(x_i, \boldsymbol{x}_{-i}) := -q'(x_i)\Big\{\mathrm{E}\Big[b\Big(w-\pi-\min\big(\frac{\ell B_{-i}}{n},d\big)\Big) \mid \boldsymbol{x}_{-i}\Big] - \mathrm{E}\Big[b\Big(w-\pi-\min\big(\frac{\ell+\ell B_{-i}}{n},d\big)\Big) \mid \boldsymbol{x}_{-i}\Big]\Big\} \tag{4}$$

It can be verified that $\delta(x_i, \boldsymbol{x}_{-i}) \geq 0$ and $\lim_{n\to\infty} \delta(x_i, \boldsymbol{x}_{-i}) = 0$ for all $i \in \mathcal{M}_n$ (see Appendix). Employing the notation defined in equation (4), we obtain:

$$\begin{aligned} \frac{\partial u_i(x_i, \boldsymbol{x}_{-i}; \mathbf{G})}{\partial x_i} &= (1+\sum_j g_{ij})\delta(x_i, \boldsymbol{x}_{-i}) - c'(x_i) \\ &= \underbrace{\delta(x_i, \boldsymbol{x}_{-i})}_{\text{marginal reciprocal altruism}} + \underbrace{\textstyle\sum_j g_{ij}\delta(x_i, \boldsymbol{x}_{-i})}_{\text{marginal pure altruism}} - \underbrace{c'(x_i)}_{\text{marginal effort cost}} \end{aligned} \tag{5}$$

Here, we distinguish between pure altruism and reciprocal altruism in participants' motivations. This distinction is inspired by Andreoni (1989), where the author highlights two benefits of contributing to public goods: enjoying the group's contributions (reciprocal altruism) and experiencing a warm glow from giving (pure altruism). Reciprocal altruism involves financial benefits from reduced shared losses, while pure altruism refers to genuine concern for friends' financial gains from one's higher effort. Pure altruism exists only if at least one participant is connected in the social network ($\mathbf{G} \neq \mathbf{0}$), while reciprocal altruism persists even without a social network ($\mathbf{G} = \mathbf{0}$).

The analysis of equation (5) indicates that the marginal benefit of increased effort includes both reciprocal altruistic and pure altruistic aspects, while the cost of heightened effort represents the associated disutility. Furthermore, an increase in $\sum_j g_{ij}$ amplifies the marginal pure altruistic benefit, thereby motivating a representative participant i to intensify her effort.

4.2 Existence of equilibrium

After formulating the problem of the effort equilibrium, we commence our investigation by examining its existence. This inquiry stands as a crucial prerequisite for further analysis of equilibrium properties. The ensuing proposition establishes that there exists at least one solution, denoted as $\boldsymbol{x}^* = (x_1^*, \ldots, x_n^*)$, satisfying the condition (3), thereby ensuring the existence of the effort equilibrium.

Proposition 1 (Existence of Nash equilibrium) *For a P2P insurance pool with a social network* $\mathbf{G}$*, there exists at least one Nash equilibrium* $\boldsymbol{x}^*(\mathbf{G})$ *with continuous effort.*

Proof: From equations (4) and (5), we observe that $\frac{\partial u_i(x_i, \boldsymbol{x}_{-i}; \mathbf{G})}{\partial x_i}$ is continuous in $\boldsymbol{x}$. Moreover, it is also strictly decreasing with respect to x_i for each fixed value of $\boldsymbol{x}_{-i}$ because $\frac{\partial^2 u_i(x_i, \boldsymbol{x}_{-i}; \mathbf{G})}{\partial x_i^2} < 0$. Consequently, there exists a unique point $x_i^* > 0$ at which $\frac{\partial u_i(x_i, \boldsymbol{x}_{-i}; \mathbf{G})}{\partial x_i}|_{x_i = x_i^*} = 0$, or $x_i = 0$ at which $\frac{\partial u_i(x_i, \boldsymbol{x}_{-i}; \mathbf{G})}{\partial x_i}|_{x_i = 0} \leq 0$. This implies the absence of local maxima, thereby indicating a unique best response of participant i that varies continuously in $\boldsymbol{x}_{-i}$. The conclusion follows directly from Kakutani's theorem (Kakutani 1941). ■

After Proposition 1 confirms the existence of at least one Nash equilibrium with continuous effort, we then investigate a method for searching and analyzing the equilibrium set. In the following section, we explore the equivalent condition for reaching Nash equilibria.

4.3 Equivalent condition of equilibrium

The determination of effort at equilibrium involves each participant continuously exerting effort until the marginal benefit, represented by $(1+\sum_j g_{ij})\delta(x_i, \boldsymbol{x}_{-i})$, equals the marginal cost of effort, denoted by $c'(x_i)$. However, the dynamics of interactions among participants and the possible existence of multiple equilibria in P2P insurance introduce complexity into the equilibrium analysis. In the following, we provide an equivalent condition for solving equilibrium in a P2P insurance pool.

Let $\boldsymbol{x}^*(\mathbf{G})$ be the participants' effort vector at a Nash equilibrium. The subsequent proposition provides the complete set of Nash equilibria for any social network $\mathbf{G}$. For simplicity, we do not distinguish $\boldsymbol{x}^*(\mathbf{G})$ and $\boldsymbol{x}^*$ when no confusion arises.

Proposition 2 (Identification of Nash equilibria) *For a P2P insurance pool with social network* $\mathbf{G}$*,* $\boldsymbol{x}^*(\mathbf{G})$ *is a Nash equilibrium if and only if the following (in)equalities hold for all* $i \in \mathcal{M}_n$*:*

1. $(1 + \sum_j g_{ij})\delta(x_i^*, \boldsymbol{x}_{-i}^*) - c'(x_i^*) \leq 0$;
2. $x_i^* \geq 0$;
3. $x_i^* \left[(1 + \sum_j g_{ij})\delta(x_i^*, \boldsymbol{x}_{-i}^*) - c'(x_i^*)\right] = 0$

Proposition 2 provides a sufficient and necessary condition for Nash equilibrium. With this condition, we can theoretically obtain the equilibrium set for any social network $\mathbf{G}$. It suggests that before reaching equilibrium, each participant incrementally augments her effort until the marginal benefit of higher effort, encompassing both reciprocal altruistic and pure altruistic benefits, equals the marginal cost of effort. This condition encapsulates the essence of equilibrium attainment within the P2P insurance framework, where participants aim to optimize their personal utilities while considering their acquaintances within the social network.

With the equivalent condition in Proposition 2 for achieving the Nash equilibrium, we turn to describe the set of all Nash equilibria. In the following section, we discover a hyper-rectangle boundary containing all Nash equilibria for a given social network structure.

4.4 Hyper-rectangle boundary of Nash equilibria

For a P2P insurance arrangement characterized by a social network $\mathbf{G}$, the Nash equilibrium is denoted as $\boldsymbol{x}^*(\mathbf{G}) = (x_1^*(\mathbf{G}), \ldots, x_n^*(\mathbf{G}))$. Let $\underline{\boldsymbol{x}} = (\underline{x}_1, \ldots, \underline{x}_n)$, $\overline{\boldsymbol{x}} = (\overline{x}_1, \ldots, \overline{x}_n)$,and $[\underline{\boldsymbol{x}}, \overline{\boldsymbol{x}}] := \{(x_1, \ldots, x_n) : \underline{x}_i \leq x_i \leq \overline{x}_i, \text{ for } i = 1, \ldots, n\}$[10], then, we depict the Nash equilibrium set with a hyper-rectangle determined by a given social network $\mathbf{G}$:

Proposition 3 (Hyper-rectangle boundary of Nash equilibria) *There exists a hyper-rectangle containing all Nash equilibria, expressed as*

$$\boldsymbol{x}^*(\mathbf{G}) \in [\underline{\boldsymbol{x}}(\mathbf{G}), \overline{\boldsymbol{x}}(\mathbf{G})],$$

[10]For ease of memory, one can regard $[\underline{\boldsymbol{x}}, \overline{\boldsymbol{x}}]$ as a "cage" that locks $(x_1, \ldots, x_n)$ inside.

where $\underline{\boldsymbol{x}}(\mathbf{G})$ *and* $\overline{\boldsymbol{x}}(\mathbf{G})$ *are the smallest and largest Nash equilibrium, respectively.*

For ease of proof, we consider a common case where participants are not excessively risk-averse.[11] Despite the possibility of multiple hard-to-describe Nash equilibria within a given social network, there is a silver lining: if participants do not exhibit excessively high risk aversion, then, each participant will consistently exert more effort in response to others' high effort (referred to as "strategic complements"). This enables us to describe the Nash equilibrium set with a hyper-rectangle, which is uniquely determined by a given social network.

Proposition 3 states that for every effort equilibrium $\boldsymbol{x}^*(\mathbf{G}) = (x_1^*(\mathbf{G}), \ldots, x_n^*(\mathbf{G}))$, the effort level of each participant is bounded, represented as $\underline{x}_i(\mathbf{G}) \leq x_i^*(\mathbf{G}) \leq \overline{x}_i(\mathbf{G})$ for all $i \in \mathcal{M}_n$. Notably, the Nash equilibrium set is always non-empty because of the existence of Nash equilibria, as proven in Proposition 1 Figure 2 illustrates the hyper-rectangle boundary for a three-participant example. Specifically, the axes of x_1, x_2 and x_3 indicate the efforts of participant 1,2 and 3, respectively. $\underline{x}_i$ ($\overline{x}_i$) is the effort paid by participant i at the smallest (largest) equilibrium for $i = 1, 2$ and 3. By Proposition 3, every Nash equilibrium point $\boldsymbol{x}^*(\mathbf{G})$ lies within the rectangular cuboid.

5 Effect of social network

In this section, we explore how changes in the structures of social networks affect participants' effort strategies at equilibria. Understanding the relationship between participants' effort levels and their social connections at Nash equilibrium is a fundamental inquiry. However, addressing this question poses significant challenges due to the complex interactions inherent in P2P insurance arrangements. In the following, we explore the properties

[11]Mathematically, this assumption posits that $a(w_r) := \frac{1}{2} \cdot \frac{-b''(w_r)}{b'(w-\pi)}$, is not extremely large for $w_r \in [w-\pi-d, w-\pi]$. This condition implies that each participant's effort is an *ordinary good*. This assumption is widely adopted in economic literature because real-life examples of non-ordinary goods (also known as *Giffen goods*) are rare, see e.g. Jensen & Miller (2008) and Ehrlich & Becker (1972), and w_r only frustrates in a very small range $[w - \pi - d, w - \pi]$.

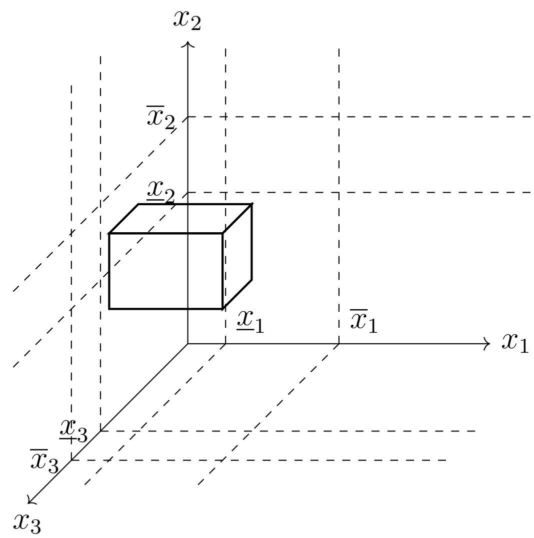

Figure 2: Hyper-rectangle for the three-dimension case

of effort equilibria and conduct comparative statics to elucidate the relationship between social connections and effort levels at Nash equilibria.

5.1 Centrality in network

A P2P insurance pool may exhibit various structures of social networks. To compare the positions of different participants within the same network $\mathbf{G}$, we first investigate the impact of centrality. Our objective is to assess the centrality of each participant and subsequently analyze its influence on her effort level at Nash equilibrium. The most straightforward approach to gauge participant i's "connected" intensity within the social network is through her degree of centrality, defined as the sum of her social connections to other participants in the same social network, denoted as $\sum_j g_{ij}$. This metric offers insight into the prominence of participant i within the network, providing a basis for understanding her relative influence and position.

The following proposition concludes that participants' efforts are positively correlated with their centralities in a social network.

Proposition 4 (Centrality and Nash equilibria) *For any Nash equilibrium* $\boldsymbol{x}^*(\mathbf{G}) = \big(x_1^*(\mathbf{G}), \ldots, x_n^*(\mathbf{G})\big)$ *associated with a given social network* $\mathbf{G}$*, if participant* j*'s central-*

ity is greater than that of participant i, that is, $\sum_k g_{jk} \geq \sum_k g_{ik}$, then it follows that participant j's effort is larger than that of participant i, that is,

$$x_j^*(\mathbf{G}) \geq x_i^*(\mathbf{G}).$$

The rationale behind Proposition 4 can be intuitively understood as follows: participants who exhibit stronger concern for a larger number of individuals within the pool are likely to experience a heightened level of marginal pure altruistic benefit. Consequently, they are more inclined to contribute greater efforts for their collective welfare. This inclination stems from their increased sense of responsibility and commitment to the well-being of other individuals in the same network. This alignment between social centrality and effort expenditure underscores the intricate dynamics between individual motivations and network structures within P2P insurance pools.

5.2 Network with higher connectivity

Proposition 4 introduces a method for comparing the equilibrium efforts of different participants within the same social network, i.e. horizontal comparison. Now, we turn to consider a more complex scenario by comparing the equilibrium effort of the same participant across different social networks, i.e. vertical comparison.

Let us consider two social networks denoted as $\mathbf{G}$ and $\mathbf{G}'$. We define that network $\mathbf{G}'$ is more "connected" than network $\mathbf{G}$, if the social connection between any pair i and j in network $\mathbf{G}'$ surpasses that in network $\mathbf{G}$, i.e., $\mathbf{G}' \geq \mathbf{G}$ (which signifies $g'_{ij} \geq g_{ij}$ for any i and j). For example, a straightforward scenario illustrating $\mathbf{G}' \geq \mathbf{G}$ is when $\mathbf{G}$ is a sub-graph of $\mathbf{G}'$, as in this scenario $\mathbf{G}'$ encompasses more connections than $\mathbf{G}$.

Recall that $\overline{\boldsymbol{x}}(\mathbf{G})$ and $\underline{\boldsymbol{x}}(\mathbf{G})$ denote the largest and smallest Nash equilibrium respectively. Let $\boldsymbol{x} \geq \boldsymbol{y}$ indicate $x_i \geq y_i$ for all $i \in \mathcal{M}_n$, the following proposition affirms that participants consistently increase their efforts within a more connected network, in both scenarios of the largest and smallest Nash equilibria.

Proposition 5 (More connected network and Nash equilibria) *When $\mathbf{G}' \geq \mathbf{G}$, we have $\overline{\boldsymbol{x}}(\mathbf{G}') \geq \overline{\boldsymbol{x}}(\mathbf{G})$ and $\underline{\boldsymbol{x}}(\mathbf{G}') \geq \underline{\boldsymbol{x}}(\mathbf{G})$. Furthermore, if both $[\underline{\boldsymbol{x}}(\mathbf{G}), \overline{\boldsymbol{x}}(\mathbf{G})]$ and*

$[\underline{\boldsymbol{x}}(\mathbf{G}'), \overline{\boldsymbol{x}}(\mathbf{G}')]$ *are singleton sets, then the unique Nash equilibria* $\boldsymbol{x}^*(\mathbf{G})$ *and* $\boldsymbol{x}^*(\mathbf{G}')$ *satisfy* $\boldsymbol{x}^*(\mathbf{G}) \geq \boldsymbol{x}^*(\mathbf{G}')$*.*

It is worth noting that Proposition 5 only compares the participants' efforts at the largest and smallest Nash equilibria, rather than at every single Nash equilibrium. It highlights that both the upper and lower boundaries of the hyper-rectangle, encompassing all Nash equilibria, ascend in response to the heightened social connections between participants.

The underlying rationale behind this proposition is that intensified social connections prompt participants to augment their efforts (referred to as "direct effect"), driven by increased pure altruistic benefits. Then, any escalation in a participant's effort further incentivizes other participants to reciprocate this participant with their even heightened efforts (referred to as "indirect effect"), owing to strategic complementarity. This positive feedback loop continues, resulting in the expanded efforts paid by the participant and the others. Given the alignment of the direct and indirect effects of these adjustments, we anticipate that the impact of a more connected social network would be magnified.

5.3 Expansion of pool size

In most P2P insurance risk pools, the number of participants (often dozens to hundreds) is relatively small compared to traditional insurance, suggesting a potential trade-off between risk diversification and moral hazard. Although a larger pool can enhance the diversification of the pooled risk, it may exacerbate the moral hazard problem. The following proposition confirms this hypothesis: the participants' efforts tend to diminish as the pool size increases. For simplicity, we denote $\mathbf{G}_n$ as an arbitrary social network associated with n participants, which satisfies $\lim_{n\to\infty} \sum_{j=1}^{n} g_{ij} < \infty$ for all $i \in \mathcal{M}_n$.

Proposition 6 (Pool expansion and Nash equilibria) *Let* $\boldsymbol{x}^*(\mathbf{G}_n)$ *be a Nash equilibrium, then we have*

$$\lim_{n\to\infty} \boldsymbol{x}^*(\mathbf{G}_n) = \mathbf{0}$$

Notably, $\lim_{n\to\infty}\sum_{j=1}^{n} g_{ij} < \infty$ is not a strong assumption, as it implies that each participant's centrality, expressed as $\sum_{j=1}^{n} g_{ij}$ for participant i, approaches a finite value as the risk pool expands. This assumption is grounded in reality as socializing time is inherently limited.

Proposition 6 can be readily derived from the fact that the marginal reciprocal altruistic benefit resulting from a participant's "moral" behaviour is uniformly distributed among all participants. Consequently, as the number of participants increases, the reciprocal altruistic benefit of higher effort declines, i.e. $\lim_{n\to+\infty}\delta(x_i,\boldsymbol{x}_{-i}) = 0$ for all $i \in \mathcal{M}_n$, leading to a decreased willingness among participants to exert higher effort. This finding aligns with the empirical evidence presented by Guinnane & Streb (2011) as well as the theoretical studies conducted by Lee & Ligon (2001) and von Bieberstein et al. (2019). In essence, the concept of high effort resembles a public good, which may be under-supplied due to the free-rider problem.

However, because $\boldsymbol{x}^*(\mathbf{G}_n) > \mathbf{0}$ possibly happens for a finite n, Proposition 6 contradicts the conclusion put forth by Ehrlich & Becker (1972), who examined moral hazard in traditional insurance and concluded that full insurance coverage would always result in zero prevention effort to reduce loss probability. The underlying reason is that in traditional insurance, fully insured individuals transfer the entirety of their risks to insurance companies, whereas fully insured participants in P2P insurance still retain some risks that come from the shared loss.

6 Numerical illustration

6.1 Setting and calibration

In this section, we provide numerical examples using some simplified social networks. To begin with, we introduce specified function forms and their associated parameter values for numerical illustration. We assume that participants' utility functions adhere to the Constant Relative Risk Aversion (CRRA) form, characterized by $b(w) = \frac{w^{1-\gamma}}{1-\gamma}$, where w denotes wealth and γ represents the coefficient of relative risk aversion. In addition, we consider the cost of effort to be $c(x) = \frac{h}{2}x^2$, where h represents a scaling factor. Meanwhile, the loss probability function, denoted as $q(x) = \frac{q_0}{1+x}$, captures the probability of experiencing a loss, where q_0 signifies an initial probability value. These functions and parameters describe the essential dynamics within our model framework. Detailed values of the parameters utilized in our analysis are summarized in the subsequent table for clarity.

Table 1: Parameter values

Parameter	h	ℓ	w	γ	q_0	d	π
Value	1×10^{-5}	10	100	2.5	0.2	5	1

In the following section, we employ two simple networks as examples to illustrate how the equilibrium loss probability vector, denoted as

$$\boldsymbol{q}\big(\boldsymbol{x}^*(\mathbf{G})\big) := \Big(q\big(x_1^*(\mathbf{G})\big), \ldots, q\big(x_n^*(\mathbf{G})\big)\Big),$$

varies with the social network $\mathbf{G}$.[12] The simulation results reveal that the Nash equilibrium happens to be unique, i.e. $\underline{\boldsymbol{x}}(\mathbf{G}) = \overline{\boldsymbol{x}}(\mathbf{G})$, indicating that the hyper-rectangle, i.e. $[\underline{\boldsymbol{x}}(\mathbf{G}), \overline{\boldsymbol{x}}(\mathbf{G})]$, is a singleton. Both numerical examples to be illustrated in the following section have a unique equilibrium.

6.2 Examples of network structures

In this section, we first apply a canonical star network to illustrate the variation of the equilibrium loss probability vector, denoted as $\boldsymbol{q}\big(\boldsymbol{x}^*(\mathbf{G})\big)$, in response to the changes in the social network $\mathbf{G}$. Subsequently, using a circular network as an example, we demonstrate that participants' efforts converge to zero with the expansion of the risk pool, i.e. $\lim_{n\to+\infty} \boldsymbol{q}\big(\boldsymbol{x}^*(\mathbf{G})\big) = \boldsymbol{q}\big(\mathbf{0}\big)$.

[12] We provide the algorithm for numerically deriving Nash equilibria in the Appendix.

6.2.1 Star network

We consider P2P insurance within a simplified star network comprising four participants, as depicted in Figure 3. The figure illustrates the star network's configuration, where the nodes ①, ②, ③, and ④ denote individual participants, and the curved, directed, and weighted arrows denote their social connections. In this social arrangement, each participant except ④ only maintains a social connection with a centralized participant ④. This forms a star-like network structure, where participants lack direct connections with each other. Specifically, their social connections in the star network can be represented by the following matrix:

$$\mathbf{G}^{\text{Star}} = \begin{bmatrix} 0 & 0 & 0 & 1g \\ 0 & 0 & 0 & 2g \\ 0 & 0 & 0 & 3g \\ 1g & 2g & 3g & 0 \end{bmatrix}$$

Here, the connection strength between any two participants varies and is measured by a unit of g. The parameter g signifies the intensity of interactions, understood as the frequency of gatherings, such as family reunions, team-building activities for colleagues, or social events among friends. The differing strengths of connections between participants shape the dynamics of the effort that each participant puts forth within the star network.

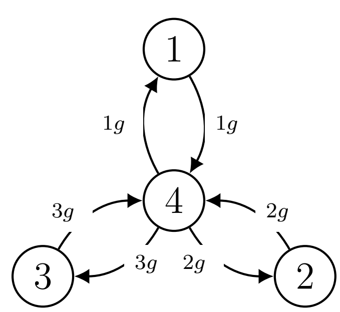

Figure 3: Star network

Figure 4 illustrates the variation in the equilibrium loss probability of each participant in response to the changes in g. We can observe a notable decrease in each participant's loss probability as g increases. This decline is attributable to both direct and indirect

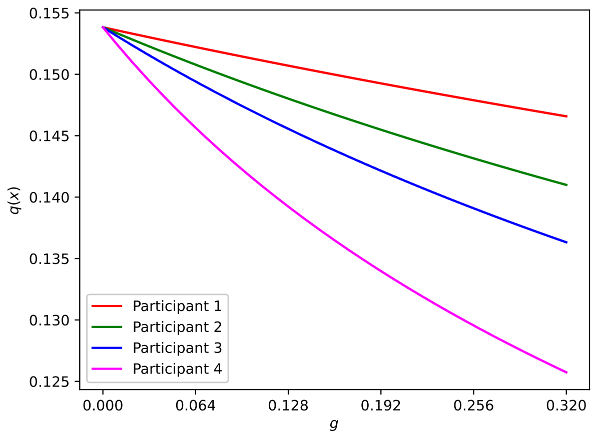

Figure 4: g and equilibrium loss probabilities

effects. Firstly, the strengthened relationships among participants incentivize greater effort due to pure altruistic motivations, representing the direct effect. Secondly, the increased effort from participants further stimulates additional exertion due to strategic complements, also known as the indirect effect. This finding corroborates the insight presented in Proposition 5.

6.2.2 Circle network

We further explore the example of the circle network, depicted in Figure 5. Similarly,

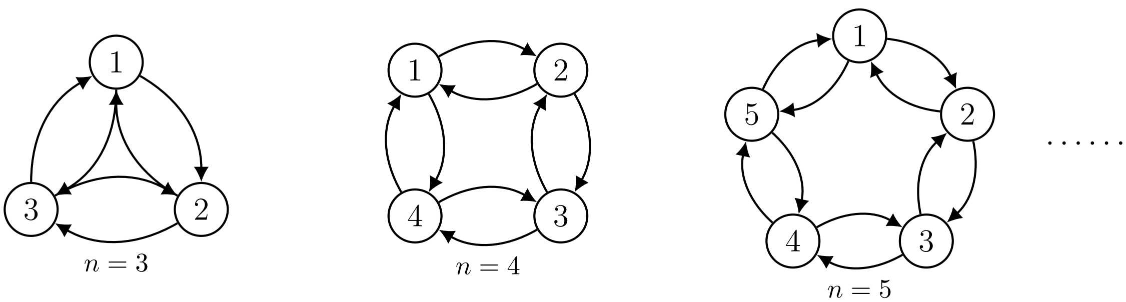

Figure 5: Increasingly Expanded Circle network

the nodes in the figure represent individual participants, while the curved, directed, but unweighted arrows denote their constant social connections. In the circle network, the participants are arranged in a circle, with each having two neighbours——one to the left and the other to the right. The circle network is also known as the local interaction network (Bramoullé et al. 2014). The circle network with n participants can be represented by an $n \times n$ matrix

$$\mathbf{G}_n^{\text{Circle}} = \begin{bmatrix} 0 & 0.5 & 0 & \cdots & 0 & 0.5 \\ 0.5 & 0 & 0.5 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0.5 & 0 & 0 & \cdots & 0.5 & 0 \end{bmatrix}_{n \times n}$$

Here, regardless of the number of participants in the circle network, i.e. the size of n in $\mathbf{G}_n^{\text{Circle}}$, each participant maintains a constant social connection of $g = 0.5$ with their two immediate neighbours. Thus, the social network among participants exhibits a circle-like network structure.

We further investigate the variations in equilibrium loss probabilities concerning the expansion of the circle network with n participants. Specifically, we examine increasingly expanding circle networks for $n = 3, 4, \ldots, 300$ participants. Figure 6 demonstrates how the equilibrium loss probabilities of participants vary with respect to the number of participants.

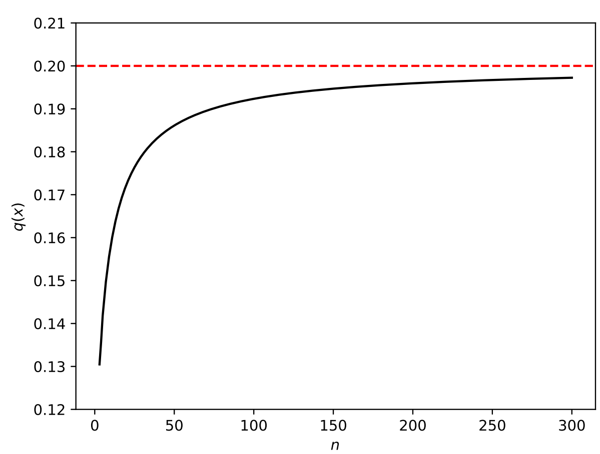

Figure 6: Loss probabilities in increasingly expanded circle network

Notably, due to homogeneity, each participant exerts the same effort, resulting in an identical loss probability. Figure 6 shows that this identical loss probability increases with the expanded pool size at a decelerating rate. From 13.05% ($n = 3$) to 18.61%

($n = 50$), and subsequently to 19.72% ($n = 300$), the loss probability escalates, approximating 20% as $n \to \infty$. This observation aligns with Proposition 6, which posits that $\lim_{n\to\infty} \boldsymbol{q}\big(\boldsymbol{x}^*(\mathbf{G})\big) = \boldsymbol{q}(\mathbf{0})$ (recall that $q(0) = 20\%$). The underlying rationale for this phenomenon lies in participants' limited gains from higher efforts in a larger pool, denoted as $\lim_{n\to\infty} \delta(x_i, \boldsymbol{x}_{-i}) = 0$ for all $i \in \mathcal{M}_n$, while still bearing the full cost of effort individually.

7 Conclusion

P2P insurance, an emerging phenomenon of the InsurTech industry worldwide, has been providing more affordable insurance coverage for communities of participants. We analyze the issue of moral hazard associated with the framework of P2P insurance from the theoretical perspective. Specifically, our model provides an answer to the crucial question of how the social network within the community affects the participants' incentive to spend effort in precautionary loss prevention.

This study carries some main contributions. Firstly, we introduce a quantitative framework that considers the interplay between social networks and moral hazard in P2P insurance, allowing for a comprehensive analysis of risk pooling among acquainted participants. Secondly, our investigation into the equilibria of participants' efforts reveals intriguing insights. We find that efforts in P2P insurance exhibit strategic complementarity, particularly when participants are not excessively risk-averse. Our analysis demonstrates the existence of Nash equilibria regarding effort, wherein participants with stronger social connections tend to exert greater effort. Notably, all Nash equilibria are contained within a hyper-rectangle, with the boundaries representing the smallest and largest equilibrium points. Lastly, we delve into the impact of social network structures on participants' behaviour at equilibria. Our findings suggest that as social connections between participants strengthen, efforts at the smallest and largest equilibria intensify. Moreover, we observe a tendency for participants' effort exertion to diminish as the size of the risk pool expands. In sum, our theoretical analysis offers valuable insights into the moral hazard in P2P insurance, shedding light on both its underlying mechanisms and potential strategies for mitigation.

Nevertheless, this study has some limitations, prompting the exploration of potential avenues for further research. While our investigation delves into the moral hazard problem within exogenous social networks, deposits, and insurance premiums, future studies may delve into the endogenous nature of these variables. Furthermore, the scope of our findings may be broadened to encompass alternative risk-sharing frameworks, including those characterized by the co-monotonicity property.

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A Appendix: Algorithm for numerical illustration

In this appendix, we present the algorithm used to numerically identify the hyper-rectangle containing all Nash equilibria, i.e., $\boldsymbol{x}^*(\mathbf{G}) \in [\underline{\boldsymbol{x}}(\mathbf{G}), \overline{\boldsymbol{x}}(\mathbf{G})]$.

The following procedure describes the algorithm to find the hyper-rectangle of the Nash equilibria, i.e., $\boldsymbol{x}^*(\mathbf{G}) \in [\underline{\boldsymbol{x}}(\mathbf{G}), \overline{\boldsymbol{x}}(\mathbf{G})]$. Initially, we discretize $\mathbb{R}_+$ and generate a space $\mathcal{X} = \{\frac{m}{r} : m \in \mathcal{Z}_+\}$ where $r = 1 \times 10^6$ represents "discretization error". This discretization ensures that the algorithm will converge to the "extremal" equilibrium, i.e., $\underline{\boldsymbol{x}}(\mathbf{G})$ or $\overline{\boldsymbol{x}}(\mathbf{G})$, in finite iteration steps. Then, we begin with all participants playing the maximal effort $\boldsymbol{x}^0 = x_{\max}\mathbf{1}$, let

$$x_i^1 = \arg\max_{x_i \in \mathcal{X}} u_i(x_i, \boldsymbol{x}_{-i}^0; \mathbf{G}), \quad \text{for all } i \in \mathcal{M}_n$$

be the best response for each $i \in \mathcal{M}_n$ at stage 1, and iteratively, let

$$x_i^k = \arg\max_{x_i \in \mathcal{X}} u_i(x_i, \boldsymbol{x}_{-i}^{k-1}; \mathbf{G}), \quad \text{for all } k \geq 2 \text{ and } i \in \mathcal{M}_n$$

be the beset response for each $i \in \mathcal{M}_n$ at stage $k \geq 2$. It follows that the point such that $\boldsymbol{x}^k = \boldsymbol{x}^{k-1}$ is very close to the largest Nash equilibrium point, i.e., $\boldsymbol{x}^k \approx \overline{\boldsymbol{x}}(\mathbf{G})$ by adjusting r. Analogously, starting from the minimal effort $\boldsymbol{x}^0 = x_{\min}\mathbf{1}$ and iterating upward, one can find the smallest Nash equilibrium point, i.e. $\underline{\boldsymbol{x}}(\mathbf{G})$ (Young & Zamir 2014, page 104).

We employ a star network as a straightforward example to illustrate the iteration process. Considering a star network depicted in Figure 7 where the bracket $\big(x_i^*, q(x_i^*)\big)$ beside participant i indicates her equilibrium effort level and its associated equilibrium loss probability. The curved, directed and weighted arrows connecting them refer to the constant degrees of social connections. we aim to introduce the iterative process. We denote a loss probability vector as $\boldsymbol{q}(\boldsymbol{x}) = [q(x_i)]_{i \in \mathcal{M}n}$. Iterating on the best response dynamics from the minimal effort, i.e., $\boldsymbol{q}(\boldsymbol{x}^0 = \mathbf{0}) = q_0\mathbf{1}$, leads to the convergence in the smallest equilibrium $\boldsymbol{q}\big(\underline{\boldsymbol{x}}(\mathbf{G})\big)$. Conversely, initiating the iteration of best response dynamics from the maximal effort, i.e., $\boldsymbol{q}(\boldsymbol{x}^0 = x_{\max}\mathbf{1}) \approx \mathbf{0}$ (where $x_{\max}$ can be any sufficiently large but finite value, and we set $x_{\max} = 1 \times 10^5$ in our simulation), results in the convergence towards the largest equilibrium $\boldsymbol{q}\big(\overline{\boldsymbol{x}}(\mathbf{G})\big)$. Ultimately, all Nash equilibria

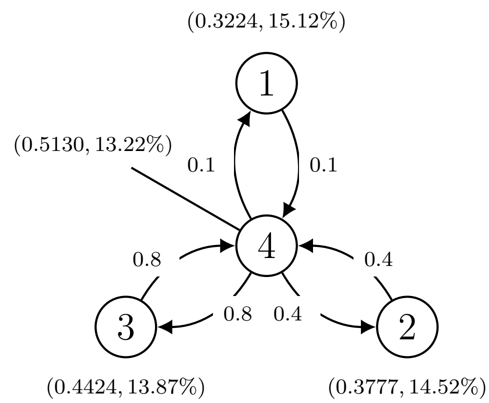

Figure 7: Star Network

lie within a hyper-rectangle, i.e., $\boldsymbol{x}^*(\mathbf{G}) \in [\underline{\boldsymbol{x}}(\mathbf{G}), \overline{\boldsymbol{x}}(\mathbf{G})]$. It is noteworthy that $\boldsymbol{q}\big(\underline{\boldsymbol{x}}(\mathbf{G})\big) \geq \boldsymbol{q}\big(\overline{\boldsymbol{x}}(\mathbf{G})\big)$, because $\underline{\boldsymbol{x}}(\mathbf{G}) \leq \overline{\boldsymbol{x}}(\mathbf{G})$ and $q(x)$ is a strictly decreasing function with respect to x.

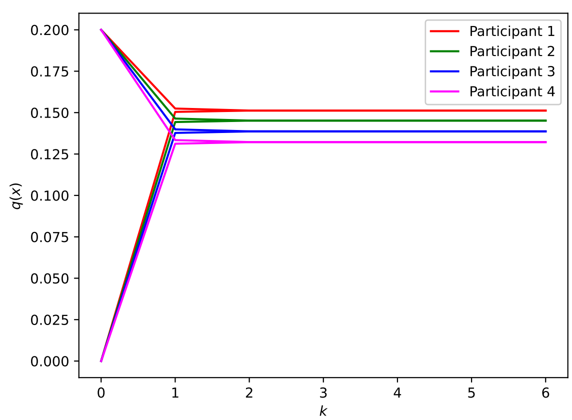

Figure 8: Iterating steps and best-response probabilities

Figure 8 illustrates the iterating process for this example. The horizontal axis represents the iteration step k, the vertical axis refers to the loss probability of participant $i = 1, 2, 3, 4$, expressed as $q(x_i^k)$, under her best-response effort x_i^k. It reveals that participants' best-response loss probability vector converges to

$$\boldsymbol{q}\big(\overline{\boldsymbol{x}}(\mathbf{G})\big) \approx \boldsymbol{q}\big(\underline{\boldsymbol{x}}(\mathbf{G})\big) = [15.12\% \ 14.52\% \ 13.87\% \ 13.22\%]^T$$

in a considerable speed. This implies the hyper-rectangle is very likely a singleton set containing a unique effort equilibrium:

$$\overline{\boldsymbol{x}}(\mathbf{G}) = \underline{\boldsymbol{x}}(\mathbf{G}) = [0.3224\ 0.3777\ 0.4424\ 0.5130]^T$$

because $q(x) = \frac{q_0}{1+x}$ is an injection, i.e. $q(x') = q(x) \implies x' = x$.

B Appendix: Proof

B.1 Proof of Proposition 2

Proof: By definition, we know that the equations

$$x_i^*(\mathbf{G}) = \arg \max_{x_i \in \mathbb{R}_+} u_i(x_i, \boldsymbol{x}_{-i}^*(\mathbf{G}); \mathbf{G})$$

for all $i \in \mathcal{M}_n$, characterize the Nash equilibria $\boldsymbol{x}^*(\mathbf{G}) = \big(x_1^*(\mathbf{G}), \dots, x_n^*(\mathbf{G})\big)$.

Because $u_i(x_i = +\infty, \boldsymbol{x}_{-i}) = -\infty$, $x_i = +\infty$ is definitely not an optimal effort for any $\boldsymbol{x}_{-i}$. Therefore, we can safely restrict the feasible set $x_i \in \mathbb{R}_+$ to $x_i \in [0, x_{\max}]$ for all $i \in \mathcal{M}_n$ where $x_{\max} < \infty$ is an arbitrarily large but finite real number. Since the feasible set $[0, x_{\max}] \subset \mathbb{R}_+$ is a non-empty, compact and convex set, and the objective function $u_i\big(x_i, \boldsymbol{x}_{-i}^*(\mathbf{G}); \mathbf{G}\big)$ is strictly concave with respect to x_i, given that others effort levels $\boldsymbol{x}_{-i} = \boldsymbol{x}_{-i}^*(\mathbf{G})$, we know that there exists a unique best-response effort $x_i^*(\mathbf{G}) = \arg\max_{x_i \in \mathbb{R}_+} u_i\big(x_i, \boldsymbol{x}_{-i}^*(\mathbf{G}); \mathbf{G}\big)$:

1. $x_i^* > 0$ and $\frac{\partial u_i\left(x_i, \boldsymbol{x}_{-i}^*; \mathbf{G}\right)}{\partial x_i}|_{x_i = x_i^*} = (1 + \sum_j g_{ij})\delta(x_i^*, \boldsymbol{x}_{-i}^*) - c'(x_i^*) = 0$ if and only if $\frac{\partial u_i\left(x_i, \boldsymbol{x}_{-i}^*; \mathbf{G}\right)}{\partial x_i}|_{x_i = 0} = (1 + \sum_j g_{ij})\delta(0, \boldsymbol{x}_{-i}^*) - c'(0) > 0$;

2. $x_i^* = 0$ if and only if $\frac{\partial u_i\left(x_i, \boldsymbol{x}_{-i}^*; \mathbf{G}\right)}{\partial x_i}|_{x_i = 0} = (1 + \sum_j g_{ij})\delta(0, \boldsymbol{x}_{-i}^*) - c'(0) \leq 0$.

■

B.2 Proof of Proposition 3

Proof: Recall that $u_i\big(x_i, \boldsymbol{x}_{-i}; \mathbf{G}\big) := v_i + \sum_j g_{ij} v_j$ (we use $u_i(x_i, \boldsymbol{x}_{-i})$ for simplicity), we define the best-response function of participant i given $\boldsymbol{x}_{-i}$ as

$$R_i(\boldsymbol{x}; \mathbf{G}) := \arg \max_{x_i \in \mathbb{R}_+} u_i\big(x_i, \boldsymbol{x}_{-i}; \mathbf{G}\big)$$

In the following, we use $R_i(\boldsymbol{x})$ for simplicity and let $\boldsymbol{R}(\boldsymbol{x}) := \big(R_1(\boldsymbol{x}), \ldots, R_n(\boldsymbol{x})\big)$. From the proof of Proposition 2, we know that $R_i(\boldsymbol{x}; \mathbf{G})$ is a function with respect to $\boldsymbol{x}$ given $\mathbf{G}$. Generally speaking, $R_i(\boldsymbol{x}; \mathbf{G})$ represents the unique best-response of i regarding $\boldsymbol{x}_{-i}$.

Denote the feasible effort (i.e. strategy) set of i as $S_i := [\underline{x}_i, \overline{x}_i]$. Let $S := [\underline{\boldsymbol{x}}, \overline{\boldsymbol{x}}]$ and $S_{-i} := [\underline{\boldsymbol{x}}_{-i}, \overline{\boldsymbol{x}}_{-i}]$. Further, we define the set of i's undominated responses to S as

$$W_i(S) := \Big\{ x_i \in S_i \mid (\exists \hat{\boldsymbol{x}} \in S)(\forall x_i' \in S_i)\big[u_i(x_i, \hat{\boldsymbol{x}}_{-i}) \geq u_i(x_i', \hat{\boldsymbol{x}}_{-i})\big] \Big\}$$

$W_i(S)$ is just the set of strategies of i that survive the process of crossing out strongly dominated strategies from S_i given others' feasible strategy set S_{-i}. We denote $\overline{\boldsymbol{W}}(S) = \big(\overline{W}_1(S), \ldots, \overline{W}_n(S)\big)$ where $\overline{W}_i(S) = \sup W_i(S)$. Let $\underline{\boldsymbol{W}}(S) = \big(\underline{W}_1(S), \ldots, \underline{W}_n(S)\big)$ where $\underline{W}_i(S) = \inf W_i(S)$. Finally, we let $W(S) := [\underline{\boldsymbol{W}}(S), \overline{\boldsymbol{W}}(S)]$. By this setting, we know that $\big(W_1(S), \ldots, W_n(S)\big) \subset W(S)$.

Step 1: Proof of $\frac{\partial^2 u_i(x_i, \boldsymbol{x}_{-i})}{\partial x_i \partial x_j} \geq 0$

$$\frac{\partial^2 u_i(x_i, \boldsymbol{x}_{-i})}{\partial x_i \partial x_j} = (1 + \sum_j g_{ij}) \frac{\partial \delta(x_i, \boldsymbol{x}_{-i})}{\partial x_j}$$

By the expression of $\delta(x_i, \boldsymbol{x}_{-i})$, we know that

$$\begin{aligned}
\frac{\partial \delta(x_i, \boldsymbol{x}_{-i})}{\partial x_j} = q'(x_i) q'(x_j) \Big\{ & \mathrm{E}\Big[b\big(w - \pi - \min(\frac{2\ell + \ell B_{-i,-j}}{n}, d)\big) \mid \boldsymbol{x}_{-i,-j} \Big] \\
& - 2\mathrm{E}\Big[b\big(w - \pi - \min(\frac{\ell + \ell B_{-i,-j}}{n}, d)\big) \mid \boldsymbol{x}_{-i,-j} \Big] \\
& + \mathrm{E}\Big[b\big(w - \pi - \min(\frac{\ell B_{-i,-j}}{n}, d)\big) \mid \boldsymbol{x}_{-i,-j} \Big] \Big\}
\end{aligned}$$

By Taylor's Theorem and Mean Value Theorem, denote $a(w_r) := \frac{1}{2} \cdot \frac{-b''(w_r)}{b'(w-\pi)}$, we have

$$b(w - \pi - z) = b(w - \pi) - b'(w - \pi) z + \frac{1}{2} \cdot b''(w - \pi - t) z^2$$

$$= b(w-\pi) - b'(w-\pi)\big[z + \frac{1}{2}\cdot\frac{-b''(w-\pi-t)}{b'(w-\pi)}z^2\big]$$
$$= b(w-\pi) - b'(w-\pi)\big[z + a(w-\pi-t)z^2\big]$$

for $z \in [0, d]$ and $t \in [0, z]$. By assumption, $a(w-\pi-t) := \frac{1}{2}\cdot\frac{-b''(w-\pi-t)}{b'(w-\pi)}$ is not extremely large, hence we can make an approximation that

$$b(w-\pi-z) \approx b(w-\pi) - b'(w-\pi)z$$

This implies that

$$\frac{\partial\delta(x_i, \boldsymbol{x}_{-i})}{\partial x_j} \approx q'(x_i)q'(x_j)b'(w-\pi)\Bigg\{\Bigg(\mathrm{E}\Bigg[\min(\frac{\ell+\ell B_{-i,-j}}{n}, d) \mid \boldsymbol{x}_{-i,-j}\Bigg] - \mathrm{E}\Bigg[\min(\frac{\ell B_{-i,-j}}{n}, d) \mid \boldsymbol{x}_{-i,-j}\Bigg]\Bigg)$$
$$- \Bigg(\mathrm{E}\Bigg[\min(\frac{2\ell+\ell B_{-i,-j}}{n}, d) \mid \boldsymbol{x}_{-i,-j}\Bigg] - \mathrm{E}\Bigg[\min(\frac{\ell+\ell B_{-i,-j}}{n}, d) \mid \boldsymbol{x}_{-i,-j}\Bigg]\Bigg)\Bigg\} \geq 0$$

Therefore, $\frac{\partial^2 u_i(x_i, \boldsymbol{x}_{-i})}{\partial x_i \partial x_j} \geq 0$, implying that participants' efforts are strategic complements.

Step 2: Proof of $W(S) = [\boldsymbol{R}(\underline{\boldsymbol{x}}), \boldsymbol{R}(\overline{\boldsymbol{x}})]$

Firstly, we prove that $[\boldsymbol{R}(\underline{\boldsymbol{x}}), \boldsymbol{R}(\overline{\boldsymbol{x}})] \subset W(S)$. By definition, we have $\boldsymbol{R}(\underline{\boldsymbol{x}}), \boldsymbol{R}(\overline{\boldsymbol{x}}) \in W(S) = W([\underline{\boldsymbol{x}}, \overline{\boldsymbol{x}}])$. Moreover, we have to show that $\boldsymbol{R}(\underline{\boldsymbol{x}}) \leq \boldsymbol{R}(\overline{\boldsymbol{x}})$, i.e. $R_i(\underline{\boldsymbol{x}}) \leq R_i(\overline{\boldsymbol{x}})$ for all $i \in \mathcal{M}_n$. In Step 1, we have proven that $\delta(x_i, \boldsymbol{x}_{-i})$ is increasing with respect to $\boldsymbol{x}_{-i}$. We assert that $\delta(x_i, \boldsymbol{x}_{-i})$ is decreasing with respect to x_i because

$$\frac{\partial\delta(x_i, \boldsymbol{x}_{-i})}{\partial x_i} = q''(x_i)\Bigg(\mathrm{E}\Bigg[b(w-\pi-\min(\frac{\ell+\ell B_{-i}}{n}, d) \mid \boldsymbol{x}_{-i}\Bigg]$$
$$- \mathrm{E}\Bigg[b(w-\pi-\min(\frac{\ell B_{-i}}{n}, d) \mid \boldsymbol{x}_{-i}\Bigg]\Bigg) \leq 0$$

Assume by contradiction that there exists $i \in \mathcal{M}_n$ such that $R_i(\underline{\boldsymbol{x}}) > R_i(\overline{\boldsymbol{x}})$, then, we have

$$0 = (1 + \sum_j g_{ij})\delta\big(R_i(\underline{\boldsymbol{x}}), \underline{\boldsymbol{x}}_{-i}\big) - c'\big(R_i(\underline{\boldsymbol{x}})\big)$$
$$\leq (1 + \sum_j g_{ij})\delta\big(R_i(\underline{\boldsymbol{x}}), \overline{\boldsymbol{x}}_{-\boldsymbol{i}}\big) - c'\big(R_i(\underline{\boldsymbol{x}})\big)$$
$$< (1 + \sum_j g_{ij})\delta\big(R_i(\underline{\boldsymbol{x}}), \overline{\boldsymbol{x}}_{-\boldsymbol{i}}\big) - c'\big(R_i(\overline{\boldsymbol{x}})\big)$$
$$\leq (1 + \sum_j g_{ij})\delta\big(R_i(\overline{\boldsymbol{x}}), \overline{\boldsymbol{x}}_{-\boldsymbol{i}}\big) - c'\big(R_i(\overline{\boldsymbol{x}})\big) = 0$$

which contradicts the fact that $0 = 0$, hence $R_i(\underline{\boldsymbol{x}}) \leq R_i(\overline{\boldsymbol{x}})$ for all $i \in \mathcal{M}_n$. Therefore

$$[\boldsymbol{R}(\underline{\boldsymbol{x}}), \boldsymbol{R}(\overline{\boldsymbol{x}})] \subset W(S) = W([\underline{\boldsymbol{x}}, \overline{\boldsymbol{x}}])$$

Secondly, we prove that $W(S) \subset [\boldsymbol{R}(\underline{\boldsymbol{x}}), \boldsymbol{R}(\overline{\boldsymbol{x}})]$. Suppose $\boldsymbol{x} \notin [\boldsymbol{R}(\underline{\boldsymbol{x}}), \boldsymbol{R}(\overline{\boldsymbol{x}})]$, we now prove that $\boldsymbol{x} \notin W(S)$. If $\boldsymbol{x} \notin [\boldsymbol{R}(\underline{\boldsymbol{x}}), \boldsymbol{R}(\overline{\boldsymbol{x}})]$, we know that there exists $i \in \mathcal{M}_n$ such that either (i) $x_i > R_i(\overline{\boldsymbol{x}})$, or (ii) $x_i < R_i(\underline{\boldsymbol{x}})$. If (i) holds, then, for an arbitrary $\boldsymbol{x} \in S = [\underline{\boldsymbol{x}}, \overline{\boldsymbol{x}}]$, we have

$$u_i(x_i, \boldsymbol{x}_{-i}) - u_i\big(R_i(\overline{\boldsymbol{x}}), \boldsymbol{x}_{-i}\big) \leq u_i(x_i, \overline{\boldsymbol{x}}_{-i}) - u_i(R_i(\overline{\boldsymbol{x}}), \overline{\boldsymbol{x}}_{-i}) < 0$$

where the first inequality uses the property of strategic complements [13], the second inequality uses the optimality of $R_i(\overline{\boldsymbol{x}})$ given $\overline{\boldsymbol{x}}_{-i}$. Hence, we know that $R_i(\overline{\boldsymbol{x}})$ strongly dominates x_i. Similarly, if (ii) holds, then, $R_i(\underline{\boldsymbol{x}})$ strongly dominates x_i. Because either (i) $R_i(\overline{\boldsymbol{x}})$ strongly dominates x_i, or (ii) $R_i(\underline{\boldsymbol{x}})$ strongly dominates x_i, recall that $\boldsymbol{R}(\underline{\boldsymbol{x}}), \boldsymbol{R}(\overline{\boldsymbol{x}}) \in W(S)$, we can infer that $\boldsymbol{x} \notin W(S)$. Therefore, we know $W(S) = [\boldsymbol{R}(\underline{\boldsymbol{x}}), \boldsymbol{R}(\overline{\boldsymbol{x}})]$.

Step 3: Proof that all Nash equilibria lie in $[\underline{\boldsymbol{x}}(\mathbf{G}), \overline{\boldsymbol{x}}(\mathbf{G})]$

Because $u_i(x_i = +\infty, \boldsymbol{x}_{-i}) = -\infty$, $x_i = +\infty$ is definitely not an optimal effort for any $\boldsymbol{x}_{-i}$. Therefore, we can safely restrict the feasible set $x_i \in \mathbb{R}_+$ to $x_i \in [0, x_{\max}]$ for all $i \in \mathcal{M}_n$ where $x_{\max} < \infty$ is an arbitrarily large but finite real number. This makes $\mathbf{0} \leq \boldsymbol{x} \leq x_{\max}\mathbf{1}$. We define $W^j([\mathbf{0}, x_{\max}\mathbf{1}]) := W\big(W^{j-1}([\mathbf{0}, x_{\max}\mathbf{1}])\big)$ with $W^0([\mathbf{0}, x_{\max}\mathbf{1}]) = [\mathbf{0}, x_{\max}\mathbf{1}]$. Similarly, let $\boldsymbol{R}^j(\mathbf{0}) := \boldsymbol{R}\big(\boldsymbol{R}^{j-1}(\mathbf{0})\big)$ and $\boldsymbol{R}^j(x_{\max}\mathbf{1}) := \boldsymbol{R}\big(\boldsymbol{R}^{j-1}(x_{\max}\mathbf{1})\big)$ with $\boldsymbol{R}^0(\mathbf{0}) = \mathbf{0}$ and $\boldsymbol{R}^0(x_{\max}\mathbf{1}) = x_{\max}\mathbf{1}$. Then, using the conclusion that $W([\underline{\boldsymbol{x}}, \overline{\boldsymbol{x}}]) = [\boldsymbol{R}(\underline{\boldsymbol{x}}), \boldsymbol{R}(\overline{\boldsymbol{x}})]$ proven in Step 2, we have

$$\begin{aligned}
W([\mathbf{0}, x_{\max}\mathbf{1}]) &= [\boldsymbol{R}(\mathbf{0}), \boldsymbol{R}(x_{\max}\mathbf{1})] \\
W^2\big([\mathbf{0}, x_{\max}\mathbf{1}]\big) &= W\big(W([\mathbf{0}, x_{\max}\mathbf{1}])\big) = W\big([\boldsymbol{R}(\mathbf{0}), \boldsymbol{R}(x_{\max}\mathbf{1})]\big) = [\boldsymbol{R}^2(\mathbf{0}), \boldsymbol{R}^2(x_{\max}\mathbf{1})] \\
&\vdots \\
W^j([\mathbf{0}, x_{\max}\mathbf{1}]) &= [\boldsymbol{R}^j(\mathbf{0}), \boldsymbol{R}^j(x_{\max}\mathbf{1})]
\end{aligned}$$

[13] Using $\frac{\partial^2 u_i(x_i, \boldsymbol{x}_{-i})}{\partial x_i \partial x_j} \geq 0$, we have

$$u_i(x_i, \boldsymbol{x}_{-i}) - u_i\big(R_i(\overline{\boldsymbol{x}}), \boldsymbol{x}_{-i}\big) = \int_{R_i(\overline{\boldsymbol{x}})}^{x_i} u_i'(z, \boldsymbol{x}_{-i})dz \leq \int_{R_i(\overline{\boldsymbol{x}})}^{x_i} u_i'(z, \overline{\boldsymbol{x}}_{-i})dz = u_i(x_i, \overline{\boldsymbol{x}}_{-i}) - u_i(R_i(\overline{\boldsymbol{x}}), \overline{\boldsymbol{x}}_{-i})$$

By definition, Nash equilibrium cannot be crossed out by iterated elimination of strongly dominated strategies, so all Nash equilibria lie in $W^j([\mathbf{0}, x_{\max}\mathbf{1}]) = [\boldsymbol{R}^j(\mathbf{0}), \boldsymbol{R}^j(x_{\max}\mathbf{1})]$. Moreover, since $\boldsymbol{R}(\cdot)$ is an increasing function with respect to $\boldsymbol{x}$ because of strategic complements, we can infer that $\{\boldsymbol{R}^j(\mathbf{0})\}_{j=1}^{\infty}$ is an increasing sequence and $\{\boldsymbol{R}^j(x_{\max}\mathbf{1})\}_{j=1}^{\infty}$ is a decreasing sequence. By the existence of the Nash equilibria, we know that $\varnothing \neq W^{\infty}([\mathbf{0}, x_{\max}\mathbf{1}]) = [\boldsymbol{R}^{\infty}(\mathbf{0}), \boldsymbol{R}^{\infty}(x_{\max}\mathbf{1})]$. If we denote $\underline{\boldsymbol{x}}(\mathbf{G}) := \boldsymbol{R}^{\infty}(\mathbf{0})$ and $\overline{\boldsymbol{x}}(\mathbf{G}) := \boldsymbol{R}^{\infty}(x_{\max}\mathbf{1})$, then , we know that $\underline{\boldsymbol{x}}(\mathbf{G})$ and $\overline{\boldsymbol{x}}(\mathbf{G})$ exist because $\mathbb{R}^n$ is complete, and Nash equilibria lie in $[\underline{\boldsymbol{x}}(\mathbf{G}), \overline{\boldsymbol{x}}(\mathbf{G})]$.

Step 4: Proof that $\underline{x}(\mathbf{G})$ and $\overline{x}(\mathbf{G})$ are the largest and smallest Nash equilibria.

We now show that $\overline{\boldsymbol{x}}(\mathbf{G})$ (and similar to $\underline{\boldsymbol{x}}(\mathbf{G})$) is a Nash equilibrium. [14] Assume by contradiction that $\overline{\boldsymbol{x}}(\mathbf{G})$ is not a Nash equilibrium, then, there exists $i \in \mathcal{M}_n$ and its associated $\hat{x}_i$ such that

$$u_i(\hat{x}_i, \overline{\boldsymbol{x}}_{-i}(\mathbf{G})) > u_i(\overline{x}_i(\mathbf{G}), \overline{\boldsymbol{x}}_{-i}(\mathbf{G}))$$

By the continuity of $u_i(\cdot, \cdot)$ with respect to x_i and $\boldsymbol{x}_{-i}$, we know that there exists some finite number j, such that

$$u_i(\hat{x}_i, \boldsymbol{R}^{j-1}(x_{\max}\mathbf{1})_{-i}) > u_i(\boldsymbol{R}_i^j(x_{\max}\mathbf{1}), \boldsymbol{R}^{j-1}(x_{\max}\mathbf{1})_{-i})$$

which contradicting the optimality of $\boldsymbol{R}_i^j(x_{\max}\mathbf{1})$.

■

B.3 Proof of Proposition 4

Proof: In the proof of Proposition 3, we know that $\delta(x_i, \boldsymbol{x}_{-i})$ is increasing with respect to $\boldsymbol{x}_{-i}$ and decreasing with respect to x_i for all $i \in \mathcal{M}_n$. We assume by contradiction that $x_j^*(\mathbf{G}) < x_i^*(\mathbf{G})$, then, we have

$$\delta\big(x_j^*(\mathbf{G}), \boldsymbol{x}_{-j}^*(\mathbf{G})\big) \geq \delta\big(x_i^*(\mathbf{G}), \boldsymbol{x}_{-j}^*(\mathbf{G})\big) \geq \delta\big(x_i^*(\mathbf{G}), \boldsymbol{x}_{-i}^*(\mathbf{G})\big)$$

[14]Because any Nash equilibrium $\boldsymbol{x}^* \in [\underline{\boldsymbol{x}}(\mathbf{G}), \overline{\boldsymbol{x}}(\mathbf{G})]$, $\overline{\boldsymbol{x}}(\mathbf{G})$ is then the largest Nash equilibrium.

The first and second inequalities are followed by the monotonicity of $\delta(\cdot,\cdot)$ with respect to x_i and $\boldsymbol{x}_{-i}$ for all $i \in \mathcal{M}_n$.

Further, we can know that

$$\begin{aligned}\frac{\partial u_j(x_j, \boldsymbol{x}_{-j}; \mathbf{G})}{\partial x_j}|_{\boldsymbol{x}=\boldsymbol{x}^*(\mathbf{G})} &= (1+\sum_k g_{jk})\delta\big(x_j^*(\mathbf{G}), \boldsymbol{x}_{-j}^*(\mathbf{G})\big) - c'\big(x_j^*(\mathbf{G})\big) \\ &> (1+\sum_k g_{jk})\delta\big(x_j^*(\mathbf{G}), \boldsymbol{x}_{-j}^*(\mathbf{G})\big) - c'\big(x_i^*(\mathbf{G})\big) \\ &\geq (1+\sum_k g_{jk})\delta\big(x_i^*(\mathbf{G}), \boldsymbol{x}_{-i}^*(\mathbf{G})\big) - c'\big(x_i^*(\mathbf{G})\big) \\ &\geq (1+\sum_k g_{ik})\delta\big(x_i^*(\mathbf{G}), \boldsymbol{x}_{-i}^*(\mathbf{G})\big) - c'\big(x_i^*(\mathbf{G})\big) \\ &= \frac{\partial u_i(x_i, \boldsymbol{x}_{-i}; \mathbf{G})}{\partial x_i}|_{\boldsymbol{x}=\boldsymbol{x}^*(\mathbf{G})}\end{aligned}$$

If $x_j^*(\mathbf{G}) > 0$, this contradicts the fact that $\frac{\partial u_j(x_j,\boldsymbol{x}_{-j};\mathbf{G})}{\partial x_j}|_{\boldsymbol{x}=\boldsymbol{x}^*(\mathbf{G})} = \frac{\partial u_i(x_i,\boldsymbol{x}_{-i};\mathbf{G})}{\partial x_i}|_{\boldsymbol{x}=\boldsymbol{x}^*(\mathbf{G})} = 0$. On the other hand, if $x_j^*(\mathbf{G}) = 0$, this contradicts the fact that $\frac{\partial u_j(x_j,\boldsymbol{x}_{-j};\mathbf{G})}{\partial x_j}|_{\boldsymbol{x}=\boldsymbol{x}^*(\mathbf{G})} \leq 0 = \frac{\partial u_i(x_i,\boldsymbol{x}_{-i};\mathbf{G})}{\partial x_i}|_{\boldsymbol{x}=\boldsymbol{x}^*(\mathbf{G})}$. Therefore, we have $x_j^*(\mathbf{G}) \geq x_i^*(\mathbf{G})$.

■

B.4 Proof of Proposition 5

Proof: In the proof of Proposition 2, we know that $R_i(\boldsymbol{x}; \mathbf{G})$ is defined as:

1. $R_i(\boldsymbol{x}; \mathbf{G}) = 0$ if and only if $\frac{\partial u_i(x_i,\boldsymbol{x}_{-i};\mathbf{G})}{\partial x_i}|_{x_i=0} \leq 0$;

2. $R_i(\boldsymbol{x}; \mathbf{G}) > 0$ and $\frac{\partial u_i(x_i,\boldsymbol{x}_{-i};\mathbf{G})}{\partial x_i}|_{x_i=R_i(\boldsymbol{x};\mathbf{G})} = 0$ if and only if $\frac{\partial u_i(x_i,\boldsymbol{x}_{-i};\mathbf{G})}{\partial x_i}|_{x_i=0} > 0$.

If $R_i(\boldsymbol{x}; \mathbf{G}) > 0$, then, by the implicit function theorem, we have

$$\frac{\partial R_i(\boldsymbol{x}; \mathbf{G})}{\partial x_j} = -\frac{\partial^2 u_i(x_i, \boldsymbol{x}_{-i})/\partial x_i \partial x_j}{\partial^2 u_i(x_i, \boldsymbol{x}_{-i})/\partial x_i^2}$$

and

$$\frac{\partial R_i(\boldsymbol{x}; \mathbf{G})}{\partial g_{ij}} = -\frac{\partial^2 u_i(x_i, \boldsymbol{x}_{-i})/\partial x_i \partial g_{ij}}{\partial^2 u_i(x_i, \boldsymbol{x}_{-i})/\partial x_i^2}$$

for all $j \neq i$. In the proof of Proposition 3 , we know that

$$\frac{\partial^2 u_i(x_i, \boldsymbol{x}_{-i})}{\partial x_i^2} = (1 + \sum_j g_{ij}) \frac{\partial \delta(x_i, \boldsymbol{x}_{-i})}{\partial x_i} - c''(x_i) < 0$$

and

$$\frac{\partial^2 u_i(x_i, \boldsymbol{x}_{-i})}{\partial x_i \partial x_j} = (1 + \sum_j g_{ij}) \frac{\partial \delta(x_i, \boldsymbol{x}_{-i})}{\partial x_j} \geq 0$$

hence $\frac{\partial R_i(\boldsymbol{x};\mathbf{G})}{\partial x_j} \geq 0$. In addition, as

$$\frac{\partial^2 u_i(x_i, \boldsymbol{x}_{-i})}{\partial x_i \partial g_{ij}} = \delta(x_i, \boldsymbol{x}_{-i}) \geq 0$$

we can infer that $\frac{\partial R_i(\boldsymbol{x};\mathbf{G})}{\partial g_{ij}} \geq 0$. Hence, $R_i(\boldsymbol{x}; \mathbf{G})$ is increasing with respect to $\boldsymbol{x}_{-i}$ and g_{ij}.

If $R_i(\boldsymbol{x}; \mathbf{G}) = 0$, then, for any $\boldsymbol{x}_{-i'} \geq \boldsymbol{x}_{-i}$ and $\mathbf{G}' \geq \mathbf{G}$,

$$R_i\big((x_i, \boldsymbol{x}_{-i'}); \mathbf{G}'\big) \geq R_i\big((x_i, \boldsymbol{x}_{-i}); \mathbf{G}\big) = 0$$

Also, $R_i(\boldsymbol{x}; \mathbf{G})$ is increasing with respcet to $\boldsymbol{x}_{-i}$ and g_{ij}. To sum up, $R_i(\boldsymbol{x}; \mathbf{G})$ is an increasing function with respect to $\boldsymbol{x}_{-i}$ and $\mathbf{G}$, regardless of whether $R_i(\boldsymbol{x}; \mathbf{G}) > 0$ or $R_i(\boldsymbol{x}; \mathbf{G}) = 0$.

As $\boldsymbol{R}(\boldsymbol{x}; \mathbf{G})$ is an increasing function with respect to $\boldsymbol{x} \in [\mathbf{0}, x_{\max}\mathbf{1}]$, [15] and $[\mathbf{0}, x_{\max}\mathbf{1}]$ is a complete lattice, by Tarskis' fixed point theorem [16], we know that $\overline{\boldsymbol{x}}(G) = \sup\{\boldsymbol{x} \in [\mathbf{0}, x_{\max}\mathbf{1}] \mid \boldsymbol{R}(\boldsymbol{x}; \mathbf{G}) \geq \boldsymbol{x}\}$ and $\underline{\boldsymbol{x}}(\mathbf{G}) = \inf\{\boldsymbol{x} \in [\mathbf{0}, x_{\max}\mathbf{1}] \mid \boldsymbol{R}(\boldsymbol{x}; \mathbf{G}) \leq \boldsymbol{x}\}$. Because $\boldsymbol{R}(\boldsymbol{x}; \mathbf{G})$ is also an increasing function with respect to $\mathbf{G}$[17], we know that

$$\{\boldsymbol{x} \in [\mathbf{0}, x_{\max}\mathbf{1}] \mid \boldsymbol{R}(\boldsymbol{x}; \mathbf{G}) \geq \boldsymbol{x}\} \subset \{\boldsymbol{x} \in [\mathbf{0}, x_{\max}\mathbf{1}] \mid \boldsymbol{R}(\boldsymbol{x}; \mathbf{G}') \geq \boldsymbol{x}\}$$

and

$$\overline{\boldsymbol{x}}(\mathbf{G}) = \sup\{\boldsymbol{x} \in [\mathbf{0}, x_{\max}\mathbf{1}] \mid \boldsymbol{R}(\boldsymbol{x}; \mathbf{G}) \geq \boldsymbol{x}\} \leq \sup\{\boldsymbol{x} \in [\mathbf{0}, x_{\max}\mathbf{1}] \mid \boldsymbol{R}(\boldsymbol{x}; \mathbf{G}') \geq \boldsymbol{x}\} = \overline{\boldsymbol{x}}(\mathbf{G}')$$

[15] That is to say, $R_i(\boldsymbol{x}; \mathbf{G})$ is an increasing function with respect to $\boldsymbol{x}_{-i}$ for all $i \in \mathcal{M}_n$.

[16] If $\mathcal{T}$ is a complete lattice and $f : \mathcal{T} \to \mathcal{T}$ is an increasing (non-decreasing) function, then, f has fixed points. Moreover, the set of fixed points of f has $\sup\{x \in \mathcal{T} : f(x) \geq x\}$ as its largest element, and $\inf\{x \in \mathcal{T} : f(x) \leq x\}$ as its smallest element, see Milgrom & Roberts (1990) for more detail.

[17] That is to say, $R_i(\boldsymbol{x}; \mathbf{G})$ is an increasing function with respect to g_{ij} for all $i \in \mathcal{M}_n$ and $j \neq i$.

Using a similar technique, we know that

$$\{\boldsymbol{x} \in [\mathbf{0}, x_{\max}\mathbf{1}] \mid \boldsymbol{R}(\boldsymbol{x}; \mathbf{G}') \leq \boldsymbol{x}\} \subset \{\boldsymbol{x} \in [\mathbf{0}, x_{\max}\mathbf{1}] \mid \boldsymbol{R}(\boldsymbol{x}; \mathbf{G}) \leq \boldsymbol{x}\}$$

and

$$\underline{\boldsymbol{x}}(\mathbf{G}) = \inf\{\boldsymbol{x} \in [\mathbf{0}, x_{\max}\mathbf{1}] \mid \boldsymbol{R}(\boldsymbol{x}; \mathbf{G}) \leq \boldsymbol{x}\} \leq \inf\{\boldsymbol{x} \in [\mathbf{0}, x_{\max}\mathbf{1}] \mid \boldsymbol{R}(\boldsymbol{x}; \mathbf{G}') \leq \boldsymbol{x}\} = \underline{\boldsymbol{x}}(\mathbf{G}')$$

Therefore, $\underline{\boldsymbol{x}}(\mathbf{G}') \geq \underline{\boldsymbol{x}}(\mathbf{G})$ and $\overline{\boldsymbol{x}}(\mathbf{G}') \geq \overline{\boldsymbol{x}}(\mathbf{G})$ for $\mathbf{G}' \geq \mathbf{G}$.

By Proposition 3, we know Nash equilibrium $\boldsymbol{x}^*(\mathbf{G}) \in [\underline{\boldsymbol{x}}(\mathbf{G}), \overline{\boldsymbol{x}}(\mathbf{G})]$ and $\boldsymbol{x}^*(\mathbf{G}') \in [\underline{\boldsymbol{x}}(\mathbf{G}'), \overline{\boldsymbol{x}}(\mathbf{G}')]$. If $[\underline{\boldsymbol{x}}(\mathbf{G}), \overline{\boldsymbol{x}}(\mathbf{G})]$ are singleton sets, then, we obviously know that $\boldsymbol{x}^*(\mathbf{G}) \geq \boldsymbol{x}^*(\mathbf{G}')$ for unique Nash equilibrium $\boldsymbol{x}^*(\mathbf{G})$ and $\boldsymbol{x}^*(\mathbf{G}')$ under $\mathbf{G}$ and $\mathbf{G}'$ respectively.

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B.5 Proof of Proposition 6

Proof: Recall that

$$\begin{aligned}\delta(x_i, \boldsymbol{x}_{-i}) := -q'(x_i)\Bigg\{&\mathrm{E}\Bigg[b\Bigg(w - \pi - \min\Big(\frac{\ell B_{-i}}{n}, d\Big)\Bigg) \mid \boldsymbol{x}_{-i}\Bigg] \\ &- \mathrm{E}\Bigg[b\Bigg(w - \pi - \min\Big(\frac{\ell + \ell B_{-i}}{n}, d\Big)\Bigg) \mid \boldsymbol{x}_{-i}\Bigg]\Bigg\}.\end{aligned}$$

We can infer that $\lim_{n\to\infty} \delta(x_i, \boldsymbol{x}_{-i}) = 0$, because

$$\Pr\left(\lim_{n\to+\infty}\left|\min\left(\frac{\ell B_{-i}}{n}, d\right) - \min\left(\frac{\ell + \ell B_{-i}}{n}, d\right)\right| = 0\right) = 1$$

Further, because $\sum_{j=1}^{\infty} g_{ij} < \infty$, we have

$$\lim_{n\to\infty}(1 + \sum_j g_{ij})\delta(x_i, \boldsymbol{x}_{-i}) = 0$$

and

$$\lim_{n\to\infty}\frac{\partial u_i(x_i, \boldsymbol{x}_{-i}; \mathbf{G}_n)}{\partial x_i} = \lim_{n\to\infty}\left[(1 + \sum_j g_{ij})\delta(x_i, \boldsymbol{x}_{-i}) - c'(x_i)\right] = -c'(x_i) < 0$$

for all $i \in \mathcal{M}_n$ and its associated x_i and $\boldsymbol{x}_{-i}$, implying that $x_i = 0$ is a strictly dominant strategy for all $i \in \mathcal{M}_n$. Therefore, $\lim_{n\to\infty} \boldsymbol{x}^*(\mathbf{G}_n) = \mathbf{0}$.

■