

Equilibrium Insurance Pricing for Large Losses^{*}

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Abstract

Catastrophe-exposed P&C insurance markets increasingly confront low-frequency but high-severity losses. When such risks interact with costly external financing, insurance pricing depends not only on expected losses but also on insurers' balance-sheet conditions and capacity constraints. In this paper, we develop a dynamic competitive equilibrium framework of insurance capacity and pricing under jump-diffusion risk and model uncertainty. We show that catastrophe jumps introduce a liability-side gamble-for-resurrection mechanism through the interaction of limited liability and recapitalization options, which can generate non-monotonic and non-smooth pricing patterns under certain conditions. Robustness concerns raise equilibrium prices and capital buffers while attenuating excessive risk-taking incentives. Using a market-disciplined calibration, we show that jump risk generates substantial pricing effects beyond expected losses alone and shapes the duration of endogenous underwriting cycles.

Keywords: Equilibrium insurance pricing; Jump risk; Model uncertainty; Liquidity management; Underwriting cycles

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1 Introduction

In recent years, property and casualty (P&C) insurance markets have faced growing exposure to low-frequency but high-severity risks arising from natural disasters, pandemics, large-scale industrial accidents, and climate-related events. Although rare, such shocks can generate outsized losses that push households, firms, and even insurers into financial distress once they occur. According to a recent report by the Swiss Re Institute, global economic losses from disaster events in 2024 reached USD 318 billion and are expected to grow at an annual rate of 5-7% in the coming years. Yet only about 43% of these losses were insured, leaving a substantial global protection gap.¹ While the key to improving market efficiency lies in pricing, the high uncertainty regarding the magnitude and timing of losses renders pricing for catastrophe insurance particularly challenging.

In this paper, we focus on two salient features that catastrophe risks display. First, large losses arrive discontinuously. Unlike business-as-usual underwriting risk, catastrophe losses cluster in rare episodes and produce large, sudden balance-sheet hits (Jaimungal and Wang, 2006; Lin et al., 2009; Chang et al., 2011). To capture these abrupt high-severity shocks, we augment the Brownian baseline with a Poisson jump component in the risk model. Second, catastrophe modeling is rife with model uncertainty, i.e., ambiguity about the underlying loss distribution (Knight, 1921; Ellsberg, 1961; Hogarth and Kunreuther, 1989). For both the continuous and jump parts, the rarity of events, limited historical data, and incomplete scientific understanding of hazard and vulnerability functions make it difficult to specify accurate statistical models. It is therefore natural for insurers to account for model uncertainty and adopt robust underwriting and pricing strategies (Berger et al., 2017; Dietz and Walker, 2019; Dietz and Niehörster, 2021).

Beyond the intrinsic characteristics of risk itself, modern insurance economics suggests that insurance pricing is also shaped by the external economic environment (Bühlmann, 1980, 1984). In particular, following the global financial crisis of 2008 when insurers' exposure to systemic risk became evident, the supply-side theory of insurance markets has received growing attention in both academia and industry (Kojen and Yogo, 2021, 2023a). Research reveals that imperfect competition, financial frictions, and regulatory constraints can impose shadow costs on insurers, which are ultimately passed on to households (Kojen and Yogo, 2015, 2016, 2022; Ge, 2022). In

¹<https://www.swissre.com/institute/research/sigma-research/sigma-2025-01-natural-catastrophes-trend.html>

particular, when external equity financing is costly, insurers' liquidity positions become valuable because they determine underwriting capacity (Cummins and Lamm-Tennant, 1994; Gron, 1994b; Cummins and Phillips, 2005; Eling et al., 2023). As a result, fluctuations in insurer capacity translate into fluctuations in insurance prices and contribute to the underwriting cycles documented in the literature (Gron, 1994a; Winter, 1994; Harrington et al., 2013).

Building on these discussions, we ask how catastrophe-related jump losses reshape equilibrium insurance pricing when insurers operate under limited liability and can restore capacity through recapitalization. In such a setting, insurance prices are determined not only by expected losses, but also by the interaction between loss discontinuities, insurers' balance sheets, and financing constraints. To address this question, we build on the dynamic equilibrium framework of Henri et al. (2016) and model a competitive insurance market in which prices are determined endogenously through market clearing. We characterize two stationary Markovian equilibria. In the benchmark equilibrium, insurers take the reference loss model as given, while in the robust equilibrium, insurers account for model uncertainty and guard against worst-case probability distortions (Hansen and Sargent, 2001, 2008; Anderson et al., 2003). Both equilibria exhibit a barrier-type structure with optimal recapitalization and dividend payout thresholds. Between these boundaries, insurance prices and market-to-book ratios are endogenously determined by the industry's aggregate capacity.

The introduction of jump risk induces significant changes in equilibrium outcomes, particularly in the capacity-driven pricing schedule. In the benchmark equilibrium, discontinuous loss shocks introduce a liability-side gamble-for-resurrection incentive that competes with the conventional capacity-scarcity effect in Henri et al. (2016). When jump losses push insurers toward the recapitalization region, limited liability and access to external capital reduce the downside consequences of aggressive underwriting while preserving the potential upside from future recovery. A projection operator is introduced to capture the endogenous recapitalization mechanism by mapping post-jump capacity back to the admissible region. The resulting price-capacity relationship can exhibit different equilibrium patterns. Depending on the severity and frequency of jump losses, the equilibrium may range from the conventional declining relationship to kinked or non-monotonic pricing patterns.

Introducing model uncertainty further modifies the equilibrium implications of jump risk. Consistent with previous empirical evidence (Kunreuther et al., 1993, 1995; Cabantous, 2007; Cabantous et al., 2011) and theoretical predictions (Dietz and Walker, 2019; Dietz and Niehörster, 2021), robust

pricing leads to higher equilibrium prices and lower underwriting scale at the same capacity level. In line with [Pang \(2026\)](#) under a Brownian risk framework, the robust equilibrium features a larger capacity range and higher market-to-book ratios, reflecting the precautionary value of liquidity under model uncertainty. With jump risk, robustness concerns modify rather than eliminate the gamble-for-resurrection incentive. By evaluating future losses under adverse probability distortions, robust insurers place greater weight on unfavorable jump realizations and become less willing to expand underwriting exposure in financially constrained states. As a result, robustness concerns attenuate excessive risk-taking incentives and reshape the price-capacity relationship by changing the relative strength of the scarcity effect and the recapitalization option.

We assess the quantitative importance of jump risk using a market-disciplined calibration based on global natural-disaster loss data. The calibration identifies approximately one major aggregate loss shock every eight years, with severe-loss events generating total losses about 2.376 times the normal annual loss scale. This calibration highlights the relevance of modeling catastrophe risk as discontinuous shocks rather than simply higher continuous volatility. Holding expected losses fixed, the introduction of jump risk substantially increases equilibrium insurance prices: in the benchmark equilibrium, the maximum loading rises from 2.86% in the matched pure-diffusion economy to 4.91% in the jump-risk economy, while the corresponding increase in the robust equilibrium is from 4.25% to 8.54%. These results show that catastrophe jump risk generates additional pricing compensation beyond expected losses alone.

We also examine how robustness concerns and financing frictions affect equilibrium outcomes. Stronger robustness concerns substantially increase insurance prices by adding compensation for uncertainty about future loss realizations. Similarly, higher external financing costs increase the shadow value of insurer capital and lead to higher equilibrium loadings. Finally, we characterize the dynamic implications of the model by studying endogenous underwriting cycles. Under the calibrated jump-risk environment, the expected cycle length is approximately 3.54 years in the benchmark equilibrium and 8.38 years in the robust equilibrium. The sensitivity of cycle duration to robustness concerns and external financing costs provides a potential explanation for why empirical estimates of underwriting cycle length vary across markets and time periods ([Boyer et al., 2012](#); [Boyer and Owadally, 2015](#)).

We contribute to the literature along several dimensions. First, we extend recent dynamic

equilibrium models of insurance capacity and pricing (Henriet et al., 2016; Luciano and Rochet, 2022). Building on this framework, Pang (2026) incorporates insurers’ concerns for model uncertainty and develops a robust equilibrium under continuous Brownian risk. Compared with these studies, we introduce catastrophe jump losses into the equilibrium framework, directly addressing the open question highlighted by Henriet et al. (2016), who suggest that Poisson risk is better suited for modeling catastrophe insurance markets.²

This extension identifies a liability-side gamble-for-resurrection incentive in insurance pricing, which has received limited attention in insurance economics. To the best of our knowledge, the closest related theoretical study is Chapter 5 of Moreno-Bromberg and Rochet (2018), which identifies a similar mechanism in insurers’ reinsurance decisions. More recently, Beyer (2025) documents state-dependent and non-monotonic risk-taking among European insurers during crisis periods, focusing on asset-side portfolio choices. In contrast, we identify a liability-side channel operating through underwriting and pricing decisions. The interaction between asset-side and liability-side risk-taking provides a promising direction for future research.

Second, we contribute to the literature on catastrophe insurance markets and insurers’ capital management. Traditional models of catastrophe insurance focus primarily on actuarial aspects, such as pricing risk transfers and modeling loss distributions (Niehaus, 2002; Born and Viscusi, 2006; Ibragimov et al., 2009; Dietz and Niehörster, 2021). However, insurers’ ability to provide coverage after severe losses depends critically on their access to risk-bearing capital. Building on earlier research emphasizing the role of capital (Jaffee and Russell, 1997; Froot and O’Connell, 1999; Froot, 2001; Zanjani, 2002; Harrington and Niehaus, 2003), we develop a structural equilibrium framework that explicitly links financing conditions to insurance pricing and capacity dynamics. More broadly, this framework contributes to the growing literature on the financial economics of insurance (Kojen and Yogo, 2021, 2023a), which emphasizes that insurers’ balance-sheet conditions and capital constraints play a central role in shaping insurance market outcomes.³

Third, we contribute to the growing literature on model uncertainty in insurance and risk management. Model uncertainty regarding the underlying data-generating process has become

²In the conclusion, Henriet et al. (2016) note that “potential avenue of research would be to consider an alternative model specification with Poisson risk, which is better suited for modeling the catastrophe insurance market.”

³See also the expanding literature on supply-side distortions and financing frictions in insurance and asset-management markets, including Kojen and Yogo (2015, 2016, 2022, 2023b); Kojen et al. (2024), Ge and Weisbach (2021); Ge (2022), Egan et al. (2022), Ellul et al. (2022), Knox and Sørensen (2024), and Kubitza (2025).

increasingly relevant in insurance and financial decision-making (Etner et al., 2012; Ilut and Schneider, 2022). Existing studies have mainly examined ambiguity aversion in insurers’ investment and reinsurance decisions (Zeng et al., 2016; Guan and Liang, 2019; Li and Young, 2019; Cheng et al., 2024), while robust insurance pricing models have often abstracted from dynamic market-clearing conditions (Zhao and Zhu, 2011; Anwar and Zheng, 2012; Pichler, 2014; Dietz and Walker, 2019). We develop a dynamic robust equilibrium framework that quantifies how model uncertainty affects insurance prices, capital management, and underwriting decisions.

The remainder of this paper is organized as follows. Section 2 introduces the model setup and defines the benchmark and robust equilibria. Sections 3 and 4 solve the two equilibria separately and analyze the implications of jump risk. Section 5 provides a market-disciplined calibration of catastrophe jump risk and examines its quantitative implications, including pricing, sensitivity analyses, and underwriting cycles. Section 6 discusses model extensions, empirical predictions, and managerial and policy implications. Finally, Section 7 concludes the paper.

2 General Model and Problem Formulation

In this section, we extend the benchmark models of Henriët et al. (2016) and Pang (2026), which respectively exclude and incorporate model uncertainty, by introducing a jump-risk component to capture instantaneous large losses. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$ be a complete probability space satisfying the usual conditions. All stochastic processes governing the insurance market in the subsequent discussion are assumed to be well-defined on this space.

2.1 Insurance Market

We consider a competitive insurance market populated by a continuum of insurers, each underwriting policies against losses that are perfectly correlated across policyholders. We abstract from idiosyncratic individual risk, since such risk is fully diversifiable in a large pool and therefore irrelevant for aggregate equilibrium. The model instead centers on non-diversifiable aggregate exposure, which is precisely the dimension most relevant for catastrophe risk.⁴ The cumulative loss process for a

⁴This modeling choice follows a common tradition in the literature on insurance market equilibrium under aggregate risk, where the central object of interest is the sector’s exposure to systematic shocks rather than policy-level heterogeneity (Henriët et al., 2016; Feng et al., 2023; Chen et al., 2025). Although Luciano and Rochet (2022) allow for insurer-specific idiosyncratic disturbances, those components are diversified away at the level of a large underwriting

representative insurer, denoted by $L \triangleq \{L_t : t \geq 0\}$, follows the dynamics:

$$dL_t = ldt + \eta dB_t + \phi dN_t, \quad (2.1)$$

where $l > 0$ represents the baseline continuous loss, $\eta > 0$ measures the insurer's exposure to systematic volatility, and $B \triangleq \{B_t : t \geq 0\}$ is a standard Brownian motion. We allow for discontinuous jump risks representing large, infrequent losses. Let $N \triangleq \{N_t : t \geq 0\}$ be a Poisson process with intensity λ , and define its compensated process as $d\tilde{N}_t = dN_t - \lambda dt$. For simplicity, we assume that all jump losses are of the same magnitude $\phi \geq 0$, and N and B are independent.⁵

Insurance contracts are assumed to be short-term, which is quite suitable for analyzing P&C insurance markets, where contracts are typically of short maturity and insurers do not accumulate long-term liabilities. Assume the insurance premium per unit of time is given by:

$$\pi_t dt = (l + \lambda\phi)(1 + p_t)dt, \quad (2.2)$$

which consists of two components: an actuarially fair premium $l + \lambda\phi$, and a loading factor p_t , which is endogenously determined through market-clearing between supply and demand. For simplicity, we refer to p_t as the equilibrium insurance price throughout the analysis.

In the literature, there are different approaches to characterize insurance market demand. Like [Henriet et al. \(2016\)](#), our theoretical framework allows for an exogenously specified, continuously differentiable demand function $D(p)$, satisfying $\frac{\partial D}{\partial p} < 0$. For greater analytical clarity, however, we adopt the approach of [Luciano and Rochet \(2022\)](#) and endogenize the demand function, which enables the equilibrium price to be determined endogenously through market clearing.

Assume that a representative insuree faces the unit loss process (2.1) and transfers a proportion β_t of the risk to the insurer by purchasing insurance at premium π_t , while retaining the remaining portfolio and do not play a first-order role in shaping market-wide equilibrium. Omitting them here therefore helps sharpen the analysis of catastrophe-related aggregate losses.

⁵In actuarial models of insurer liabilities, discontinuous losses are often represented by a compound Poisson process, as in the classical Cramér-Lundberg framework, where jump sizes are themselves random. We do not adopt that specification here, because allowing for stochastic jump magnitudes would substantially complicate the analysis. Instead, our aim is to use the simplest possible framework that still captures the effect of jump losses.

fraction $1 - \beta_t$. The insuree's instantaneous change in wealth can then be expressed as:

$$dw_t = -(1 - \beta_t)(l dt + \eta dB_t + \phi dN_t) - \beta_t(l + \lambda\phi)(1 + p_t)dt.$$

Under mean-variance preferences evaluated over an infinitesimal interval dt , the insuree chooses the optimal coverage β_t to maximize:

$$\frac{\mathbb{E}[dw_t] - \frac{\alpha}{2}\text{Var}(dw_t)}{dt} = -(1 - \beta_t)(l + \lambda\phi) - \beta_t(l + \lambda\phi)(1 + p_t) - \frac{\alpha}{2}(1 - \beta_t)^2(\eta^2 + \lambda\phi^2),$$

where $\alpha > 0$ denotes the risk-aversion parameter. Solving the first-order condition yields an interior solution:

$$\beta = 1 - \frac{l + \lambda\phi}{\alpha(\eta^2 + \lambda\phi^2)}p \triangleq D(p), \quad (2.3)$$

which is linear in p and $\frac{\partial D}{\partial p} < 0$. When $p = 0$, it corresponds to full insurance coverage. For simplicity, we temporarily abstract from the possibility that β may fall outside the economically meaningful range $[0, 1]$. We will verify this property for the equilibrium outcomes.

Since insurance contracts are assumed to have an infinitesimal duration, insurers bear no long-term liabilities and are therefore not required to hold reserves against them. On each insurer's balance sheet, the cash or liquid reserves on the asset-side, denoted by $m \triangleq \{m_t : t \geq 0\}$, should equal the equity $e \triangleq \{e_t : t \geq 0\}$. Thus, m_t simultaneously represents the book value of equity for an insurer and can also be interpreted as the net wealth of the insurer.

Suppose an insurer's underwriting scale process is given by $x \triangleq \{x_t \geq 0 : t \geq 0\}$. Over an infinitesimal period $(t, t + dt)$, the insurer collects premium income of $x_t \pi_t dt$ and incurs claim payouts of $x_t dL_t$. The difference $x_t(\pi_t dt - dL_t)$ represents the underwriting profit, which can either be retained as cash reserves or distributed as dividends. Let $\delta \triangleq \{\delta_t \geq 0 : t \geq 0\}$ denote the insurer's cumulative dividend process. The insurer optimally determines the dividend payout to distribute to shareholders. Additionally, in the event of financial distress, the insurer can raise external capital to replenish cash reserves. Let $i \triangleq \{i_t \geq 0 : t \geq 0\}$ represent the cumulative recapitalization process.

Then, the insurer's net wealth process evolves according to:

$$\begin{aligned} dm_t &= x_t(\pi_t dt - dL_t) + di_t - d\delta_t \\ &= x_t \left[\lambda\phi + (l + \lambda\phi)p_t \right] dt - x_t \eta dB_t - x_t \phi dN_t + di_t - d\delta_t. \end{aligned} \quad (2.4)$$

2.2 Optimization Objective without Model Uncertainty

Let $\mathcal{J} \triangleq [0, 1]$ denote the set of insurers, each indexed by $j \in \mathcal{J}$. Given the liquid reserve process of each individual insurer m^j , the aggregate liquid reserves (capacity) of the entire insurance sector are defined as $M \triangleq \left\{ M_t = \int_{\mathcal{J}} m_t^j dj : t \geq 0 \right\}$, which follow the process:

$$\begin{aligned} dM_t &= X_t \left[\lambda\phi + (l + \lambda\phi)p_t \right] dt - X_t \eta dB_t - X_t \phi dN_t + dI_t - d\Delta_t \\ &= D(p_t) \left[\lambda\phi + (l + \lambda\phi)p_t \right] dt - D(p_t) \eta dB_t - D(p_t) \phi dN_t + dI_t - d\Delta_t, \end{aligned} \quad (2.5)$$

where $X_t = \int_{\mathcal{J}} x_t^j dj$, $I_t = \int_{\mathcal{J}} i_t^j dj$ and $\Delta_t = \int_{\mathcal{J}} \delta_t^j dj$, represent the aggregate underwriting scale, cumulative recapitalization and dividend payouts, respectively. In equilibrium, the aggregate supply must equal market demand, leading to the market-clearing condition $X_t = D(p_t)$.

Without considering model uncertainty, we aim to derive a Markovian stationary equilibrium, where the price of insurance is a deterministic function of the aggregate level of reserves, i.e., $p_t = p(M_t)$. Each individual insurer takes the dynamics (2.5) and the equilibrium price function $p(M)$ as given and optimally chooses its underwriting scale $x \geq 0$, recapitalization policy $di \geq 0$, and dividend policy $d\delta \geq 0$ to maximize shareholder value:

$$v(m, M) = \max_{x \geq 0, d\delta \geq 0, di \geq 0} \mathbb{E} \left\{ \int_0^\infty e^{-rt} \left[d\delta_t - (1 + \gamma) di_t \right] \right\}, \quad (2.6)$$

where $r > 0$ represents the discount rate such as the risk-free rate, and $\gamma > 0$ denotes the per-unit cost of raising external capital as equity financing is costly for financial intermediaries.

Definition 1 (Benchmark Equilibrium). *In the absence of model uncertainty, a stationary Markovian competitive equilibrium consists of an aggregate liquid reserve process M , an insurance price $p(M)$, and insurance supply functions $x^j(M), j \in \mathcal{J}$, that are compatible with each insurer's optimization objective (2.6) and the market-clearing condition $\int_{\mathcal{J}} x^j(M) dj = D(p(M))$.*

2.3 Optimization Objective with Model Uncertainty

Now consider the case in which insurers are concerned about model uncertainty in the loss process, which is driven by the Brownian motion B and Poisson process N under the physical measure $\mathbb{P}$. The insurer acknowledges the existence of alternative probabilistic models and allows for distorted measures that are absolutely continuous with respect to $\mathbb{P}$ over any finite time horizon. To formalize this, let (h, g) denote a pair of density generators associated with the diffusion and jump components of the loss process, and let $\mathcal{B}$ represent the set of all such admissible pairs. For $(h, g) \in \mathcal{B}$, define the process $\xi^{h,g} \triangleq \left\{ \xi_t^{h,g} : t \geq 0 \right\}$ by:

$$\xi_t^{h,g} = \mathcal{E} \left(- \int_0^t h_s dB_s - \int_0^t g_s d\tilde{N}_s \right).$$

where $\mathcal{E}(\cdot)$ denotes the Doléans-Dade exponential of a semimartingale.

By Girsanov's Theorem, there exists a distorted probability measure $\mathbb{Q}^{h,g}$ such that $\frac{d\mathbb{Q}^{h,g}}{d\mathbb{P}} \big|_{\mathcal{F}_t} = \xi_t^{h,g}$. Under this measure, the process $B^h \triangleq \left\{ B_t^h : t \geq 0 \right\}$ defined by:

$$dB_t^h = dB_t + h_t dt,$$

is a standard Brownian motion, and the process $\tilde{N}^g \triangleq \left\{ \tilde{N}_t^g : t \geq 0 \right\}$ defined by:

$$d\tilde{N}_t^g = d\tilde{N}_t + \lambda g_t dt,$$

is a compensated Poisson process. Here, the intensity of the jump becomes $\lambda_t^g = \lambda(1 - g_t)$, which implicitly requires $g_t \leq 1$. Accordingly, the individual liquid reserve process (2.4) becomes:

$$dm_t = x_t \left[(l + \lambda\phi)p_t + \eta h_t + \lambda\phi \right] dt - x_t \eta dB_t^h - x_t \phi dN_t^g + di_t - d\delta_t.$$

The corresponding aggregate capacity dynamics are given by:

$$dM_t = D(p_t) \left[(l + \lambda\phi)p_t + \eta h_t + \lambda\phi \right] dt - D(p_t) \eta dB_t^h - D(p_t) \phi dN_t^g + dI_t - d\Delta_t.$$

Because we focus on a symmetric equilibrium, we conjecture and subsequently verify that the optimal

density generators are identical across insurers.

With model uncertainty, we adopt the variational preferences framework of [Anderson et al. \(2003\)](#) and [Maccheroni et al. \(2006a,b\)](#), and assume that each insurer's objective is given by:

$$\inf_{\mathbb{Q}^{h,g} \in \mathcal{Q}} \mathbb{E}^{\mathbb{Q}^{h,g}} \left\{ \int_0^\infty e^{-rt} \left[d\delta_t - (1 + \gamma) di_t \right] \right\} + \mathcal{K}(\mathbb{Q}^{h,g}), \quad (2.7)$$

where $\mathcal{Q}$ denotes the set of all admissible distorted probability measures $\mathbb{Q}^{h,g}$, and $\mathcal{K}(\mathbb{Q}^{h,g})$ represents the penalty term quantifying the relative entropy cost of model misspecification. In studies such as [Maenhout et al. \(2020\)](#) and [Ling et al. \(2021\)](#), the penalty term is typically specified as $\mathcal{K}(\mathbb{Q}^{h,g}) = \mathbb{E}^\mathbb{P} \left[\int_0^\infty e^{-rt} \Theta_t d\phi \left(\frac{\xi_t^{h,g}}{\xi_0^{h,g}} \right) \right]$, where $\phi(x) = x \log x$ and only Brownian risk distortions are considered. Here, we extend this formulation to include jump-risk distortions while maintaining analytical tractability. Specifically, for the Poisson component, we approximate the distortion of the compensated jump measure using the second-order expansion:

$$\lambda \left[(1 - g) \log(1 - g) + g \right] = \frac{1}{2} \lambda g^2 + \mathcal{O}(g^3) \approx \frac{1}{2} \lambda g^2.$$

This approximation preserves a quadratic penalty in the distortion terms. Accordingly, we approximate the overall penalty term by:

$$\mathcal{K}(\mathbb{Q}^{h,g}) = \frac{1}{2} \mathbb{E}^{\mathbb{Q}^{h,g}} \left[\int_0^\infty e^{-rt} \Theta_t (h_t^2 + \lambda g_t^2) dt \right],$$

which provides a quadratic approximation to the discounted, state-weighted relative entropy between $\mathbb{Q}^{h,g}$ and $\mathbb{P}$, where Θ_t governs the marginal cost of model distortion.

The insurer's value function is denoted by $v(m, M)$, jointly determined by two state variables: the individual liquid reserves m and the aggregate market capacity M . In what follows, we aim to eliminate one of these state variables to facilitate the derivation of the equilibrium solution. For tractability, we assume $\Theta_t(m_t) = \theta m_t$, where $\theta > 0$ captures the degree of robustness concern among insurers, which has two implications. First, the entropy cost becomes proportional to the insurer's liquid reserves, which is economically intuitive since larger insurers face higher uncertainty-related costs. Second, it ensures that the optimization objective (2.7) remains homogeneous in $(m, x, d\delta, di)$, implying that the value function must be linear in the insurer's net wealth.

Homogeneity implies $v(m, M) = mu(M)$, where $u(M)$ represents the common market-to-book ratio for insurers with $m > 0$. Conceptually, $u(M)$ serves as the insurance analogue of Tobin's q ratio, as discussed in [Winter \(1994\)](#) and [Henriet et al. \(2016\)](#). It reflects the market's valuation of insurers' underwriting capacity relative to their equity base.

Definition 2 (Robust Equilibrium). *In the presence of model uncertainty, a stationary Markovian competitive equilibrium consists of an aggregate liquid reserve process M , an insurance price $p(M)$, a pair of density generators $(h(M), g(M))$, and insurance supply functions $x^j(M), j \in \mathcal{J}$, that are compatible with each insurer's optimization objective (2.7) and the market-clearing condition $\int_{\mathcal{J}} x^j(M) dj = D(p(M))$.*

2.4 Barrier-Type Equilibrium

Following [Henriet et al. \(2016\)](#) and [Pang \(2026\)](#), as well as the broader literature on liquidity management, we focus on a barrier-type equilibrium solution, both without and with model uncertainty. For $o \in \{b, r\}$ (representing the benchmark and robust equilibria, respectively), let $\overline{M}^o$ and $\underline{M}^o$ denote the dividend payout and external financing boundaries, which are determined endogenously. The region between these two thresholds corresponds to the internal financing region. To ensure the existence of such a solution, we impose the following assumption.

Assumption 1. *For $o \in \{b, r\}$, we focus on equilibria that satisfy the following conditions:*

- (1) *Both $v^o(\cdot, \cdot)$ and $u^o(\cdot)$ are smooth and twice continuously differentiable.*
- (2) *For $M \in [\underline{M}^o, \overline{M}^o]$, it holds that $u^o(M) > 0$ and $u^o'(M) \leq 0$. In other words, the market-to-book ratio of the insurance sector is assumed to be a non-increasing function of aggregate capacity.*
- (3) *The payout and recapitalization boundaries satisfy $0 \leq \underline{M}^o < \overline{M}^o$. In addition, $m \geq 0$ is required, because negative liquid reserves are economically infeasible for both an individual insurer and the industry.*

These conditions simplify the equilibrium analysis and ensure a well-behaved barrier-type equilibrium. The monotonicity of $u(\cdot)$ is crucial for guaranteeing that the optimal policy takes a barrier-type form. Otherwise, the payout and recapitalization regions could become disconnected, leading to multiple thresholds and more complex strategic behavior.

3 Benchmark Equilibrium with Jump Risk

3.1 Equilibrium Characterization

In this section, we characterize the competitive equilibrium defined in Definition 1, where insurers take the reference loss model as given. Following the barrier-type equilibrium structure described in Section 2.4, insurers' optimization problem can be solved within the internal financing region, where underwriting decisions are determined by the marginal value of insurer capital. The resulting equilibrium is characterized by two endogenous functions. The first is the insurance price function $p(M)$, which determines the market-clearing loading as a function of aggregate insurance capacity. The second is the market-to-book ratio function $u(M)$, which captures the marginal value of an additional unit of insurer capital.

The following proposition summarizes the system of equations satisfied by the equilibrium. The detailed derivation based on the HJB equation and the boundary conditions is provided in Appendix A.1. We do not attempt to establish analytical existence and uniqueness of the equilibrium solution. Instead, we verify numerically that under economically reasonable parameter specifications, the system admits a stable equilibrium satisfying all required conditions.

Proposition 1 (Benchmark Equilibrium). *In the absence of model uncertainty, suppose that the following system of equations admits a solution for $p(\cdot)$ and $u(\cdot)$ on $[\underline{M}, \overline{M}]$:*

$$\begin{cases} 0 = [\lambda\phi + (l + \lambda\phi)p(M)]u(M) + D(p(M))\eta^2 u'(M) - \lambda\phi u(\Pi(M - D(p(M))\phi)), \\ ru(M) = D(p(M))[\lambda\phi + (l + \lambda\phi)p(M)]u'(M) + \frac{1}{2}D(p(M))^2\eta^2 u''(M) \\ \quad + \lambda[u(\Pi(M - D(p(M))\phi)) - u(M)], \end{cases} \quad (3.1)$$

subject to the demand function $D(p(M)) = 1 - \frac{l+\lambda\phi}{\alpha(\eta^2+\lambda\phi^2)}p(M)$, the projection operator $\Pi(x) = \min\{\overline{M}, \max\{\underline{M}, x\}\}$, and boundary conditions $\underline{M} = 0$, $u(\underline{M}) = 1 + \gamma$, $u(\overline{M}) = 1$, and $u'(\overline{M}) = 0$, together with the requirements in Assumption 1. Then, a stationary Markovian equilibrium exists:

(1) For $M \in (\underline{M}, \overline{M})$, the equilibrium insurance price is given by $p(M)$, and the common market-to-book ratio is $u(M)$.

(2) For $M \geq \overline{M}$, insurers optimally distribute dividends to shareholders until aggregate reserves fall back below $\overline{M}$. For $M \leq \underline{M}$, they raise external capital to restore reserves to $\underline{M}$.

The equilibrium characterization highlights several implications of introducing catastrophe jump risk. First, relative to the continuous-risk benchmark of [Henriet et al. \(2016\)](#), jump losses introduce discontinuous adjustments in aggregate insurance capacity. Under diffusion risk, insurers' capital evolves smoothly over time, and the relationship between capacity and pricing is determined only by the gradual accumulation or depletion of liquidity. By contrast, a catastrophe event can instantaneously reduce aggregate capacity by a large amount. Therefore, the post-jump capacity level may fall outside the admissible region in which insurers operate normally.

The projection operator $\Pi(\cdot)$ captures this endogenous recapitalization mechanism. When a catastrophe loss pushes aggregate reserves below the lower capacity boundary, insurers cannot continue underwriting with insufficient liquidity and must restore their balance sheets through external capital injection. Therefore, the projection operator is not merely a mathematical adjustment, but represents the economic constraint imposed by limited financial capacity after extreme loss events. As discussed below, this discontinuous adjustment mechanism plays a central role in shaping the equilibrium price-capacity relationship.

Second, the boundary conditions characterize insurers' optimal liquidity management policies. At the upper boundary $\overline{M}$, insurers distribute excess capital because the marginal value of retaining additional liquidity equals the opportunity cost of returning funds to shareholders, i.e., $u(\overline{M}) = 1$. At the lower boundary $\underline{M}$, insurers raise external capital because the marginal value of internal liquidity reaches the cost of external financing, i.e., $u(\underline{M}) = 1 + \gamma$. The interval $[\underline{M}, \overline{M}]$ therefore represents the endogenous capacity range supported by insurers' liquidity management decisions.

Interestingly, the endogenous recapitalization boundary here remains $\underline{M} = 0$, consistent with the continuous-risk benchmark of [Henriet et al. \(2016\)](#). This result indicates that jump risk alone does not induce insurers to maintain a strictly positive liquidity buffer before accessing external capital. Instead, with external financing costs and without model uncertainty, insurers optimally rely on recapitalization only when internal reserves are fully depleted. As shown later, introducing model uncertainty changes this mechanism by increasing the value of precautionary liquidity and altering insurers' capital management behavior.

Conditional on the existence of the equilibrium, the following property characterizes insurers' underwriting profitability.

Proposition 2. *In equilibrium, insurers' expected underwriting profits are non-negative, as the market price $p(\cdot)$ is non-negative.*

The proof of this proposition is provided in Appendix A.2. It implies that equilibrium insurance prices include a non-negative compensation for bearing catastrophe risk. This positive underwriting loading does not arise from imperfect competition, but rather reflects the cost of providing insurance capacity under aggregate jump risk and costly external financing. Therefore, equilibrium pricing incorporates not only expected losses but also the financial costs of maintaining insurance capacity.

3.2 Quantitative Analysis

In this section, we numerically solve the equilibrium system in (3.1) to characterize the quantitative implications of jump risk for equilibrium insurance pricing. The objective is not to estimate the structural parameters directly, but rather to illustrate how discontinuous loss shocks interact with insurers' balance-sheet constraints and financing frictions to shape equilibrium outcomes.

All parameters are interpreted at an annual frequency. The risk-free interest rate is set to $r = 0.04$, corresponding to a 4% annualized discount rate. Following the calibration in Luciano and Rochet (2022), we normalize the unit expected loss to $l = 1.0$ and set the volatility of continuous loss risk to $\eta = 0.28$. The demand risk-aversion parameter is set to $\alpha = 2.0$, representing a moderate degree of risk aversion among policyholders. The parameter γ captures the proportional wedge associated with external financing: raising one unit of external capital requires $(1 + \gamma)$ units of resources. We set $\gamma = 0.10$ to represent a moderate financing friction. Alternative values of γ are considered in Section 5.2 to examine the sensitivity of the equilibrium outcomes to external financing costs.

The jump component captures discontinuous loss shocks beyond the continuous fluctuation in ordinary losses. The jump intensity λ represents the annual arrival frequency of such shocks, while the jump size ϕ measures the associated loss magnitude relative to the normalized annual loss scale. Following a jump, the reduction in aggregate capacity is $D(p(M))\phi$, which depends jointly on jump severity and the equilibrium underwriting volume.

Rather than relying on a single parameterization, we consider three illustrative jump-risk environments: $(\lambda, \phi) \in \{(0.2, 2), (0.2, 0.2), (2, 0.2)\}$. The value $\lambda = 0.2$ corresponds to one jump

approximately every five years on average, whereas $\lambda = 2$ represents a deliberately high-frequency environment used to isolate the frequency channel. Similarly, $\phi = 0.2$ and $\phi = 2$ represent moderate and severe jump losses, respectively, relative to the normalized annual loss scale. These parameter combinations are intended to characterize distinct economic environments rather than to serve as point estimates of a unique empirical calibration.

For each jump-risk economy, we construct a pure-diffusion counterfactual with the same expected loss by setting $l^d = l + \lambda\phi$, $\lambda^d = 0$, and $\phi^d = 0$. This comparison holds the actuarially expected loss fixed while removing the discontinuous component of risk. Differences between the two equilibria therefore reflect the timing and variability introduced by jump losses rather than a simple difference in mean losses. Notice, in particular, that $(\lambda, \phi) = (0.2, 2)$ and $(2, 0.2)$ both satisfy $\lambda\phi = 0.4$ and hence share the same matched-mean diffusion counterfactual, $l^d = 1.4$. Comparing these two environments allows us to distinguish the effects of jump frequency from those of jump severity.

To clarify the role of the projection operator, define the unadjusted post-jump capacity as $M^J(M) \triangleq M - \phi D(p(M))$ and the projection-active region as $\mathcal{P} \triangleq \{M \in [\underline{M}, \overline{M}] : M^J(M) \leq \underline{M}\}$. For the benchmark equilibrium, $\underline{M} = 0$. Thus, when $M \in \mathcal{P}$, a jump exhausts available internal capacity and insurers must inject external capital to restore reserves to the lower boundary. When $M \notin \mathcal{P}$, the post-jump capacity remains within the internal financing region and the loss can be absorbed without immediate recapitalization.

The numerical equilibria below reveal three qualitatively different pricing regimes. Severe jumps can make the projection constraint active throughout the capacity region, eliminating any projection-induced switching point. For moderate jumps, the projection constraint becomes state dependent and generates a kink in the price schedule. Whether the resulting price-capacity relationship remains monotone then depends on the relative strength of the conventional scarcity effect and the jump-induced gamble-for-resurrection incentive. Across all three environments, the market-to-book ratio remains smooth and decreasing, consistent with Assumption 1.

3.2.1 Severe Jump Losses

We first consider the environment with $(\lambda, \phi) = (0.2, 2)$, which captures a setting in which large loss shocks occur relatively infrequently but have substantial effects on insurers' balance sheets. As shown in Figure 3.1, both the equilibrium market-to-book ratio and the equilibrium insurance price

decline smoothly with aggregate capacity.

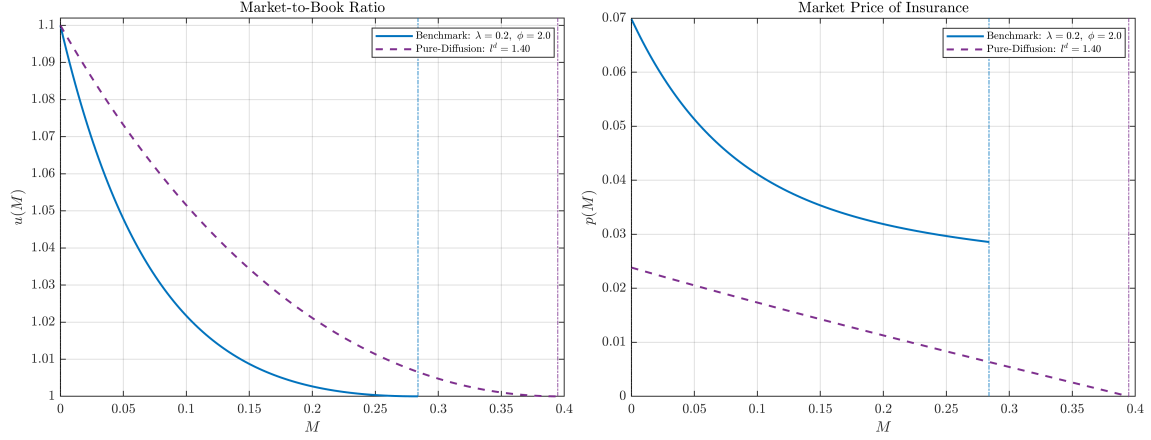


Figure 3.1: Benchmark Equilibrium under Severe Jump Losses: $(\lambda, \phi) = (0.2, 2)$.

The key feature of this environment is that the projection constraint is active throughout the admissible capacity region: $M^J(M) \leq \underline{M}$ for every $M \in [\underline{M}, \overline{M}]$. Hence, $\mathcal{P} = [\underline{M}, \overline{M}]$, and every jump exhausts insurers' internal capacity and moves the industry to the recapitalization boundary. Since the post-jump state is always located at the same boundary, the projection operator does not switch between different financing regimes. The price schedule therefore exhibits no projection-induced kink and is numerically smooth and decreasing. In this environment, the conventional capacity-scarcity effect remains dominant: greater aggregate capacity lowers the shadow cost of capital and reduces the equilibrium insurance price.

The comparison with the matched-mean diffusion benchmark is also informative. Although the two economies have the same expected loss, the jump-risk equilibrium generates a substantially higher insurance price schedule. The difference therefore cannot be attributed to expected losses alone. It reflects the additional compensation required for discontinuous loss exposure and the associated possibility of an abrupt depletion of internal capacity.

3.2.2 Frequent Moderate Jumps

We next consider the high-frequency moderate-jump environment $(\lambda, \phi) = (2, 0.2)$. This case highlights the interaction among jump risk, limited liability, and the option to recapitalize after adverse shocks. When insurers operate with limited internal capacity, a jump can move them to the recapitalization boundary. Limited liability and access to recapitalization restrict the marginal

downside of aggressive underwriting, while insurers retain the potential upside from rebuilding capital through future underwriting profits. This creates a liability-side gamble-for-resurrection incentive: financially constrained insurers may lower prices and expand underwriting exposure in an attempt to recover more quickly.

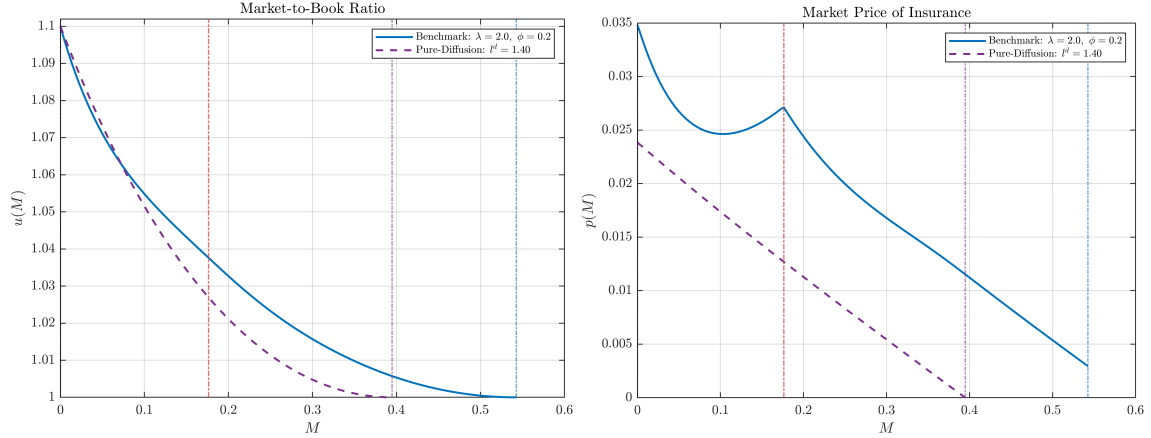


Figure 3.2: Benchmark Equilibrium under Frequent Moderate Jumps: $(\lambda, \phi) = (2, 0.2)$.

As shown in Figure 3.2, the equilibrium price function is both kinked and non-monotone. Starting from the lower capacity boundary, the insurance price initially decreases as capacity rises and reaches a local minimum around $M = 0.104$. It then increases over an intermediate capacity range before turning downward again at the projection-switching point, approximately $M_k = 0.176$.

The switching point is characterized by $M_k - \phi D(p(M_k)) = \underline{M}$. For $M < M_k$, the economy lies in the projection-active region: a jump exhausts internal capacity, and the post-jump continuation value is determined by the recapitalization boundary. For $M > M_k$, the post-jump state remains inside the internal financing region, and the continuation value varies with the remaining capacity. The kink therefore represents an endogenous transition between two post-jump financing regimes rather than a numerical artifact of the projection operator.

The non-monotonicity reflects the interaction between two competing forces. The conventional scarcity effect raises prices when aggregate capacity becomes scarce. By contrast, the gamble-for-resurrection incentive encourages insurers in financially constrained states to lower prices and expand underwriting because the recapitalization mechanism limits part of the downside while preserving the upside from recovery. Over the intermediate region, the latter force dominates and prices rise with capacity. At still lower capacity levels, however, scarcity and liquidity-preservation concerns

again become sufficiently strong, restoring the usual inverse relationship between price and capacity.

Importantly, this case has the same expected jump loss as the severe-jump environment considered above, since both satisfy $\lambda\phi = 0.4$. Nevertheless, the two economies generate sharply different pricing patterns. Severe but infrequent jumps produce an always-active projection regime with smooth and monotone pricing, whereas frequent moderate jumps generate projection switching and non-monotonicity. Thus, expected jump losses alone are not sufficient to characterize equilibrium pricing; the frequency-severity composition of jump risk matters.

3.2.3 Infrequent Moderate Jumps

Finally, we reduce the jump frequency while holding the jump size fixed, considering $(\lambda, \phi) = (0.2, 0.2)$. Comparing this environment with the preceding high-frequency case isolates the role of jump frequency in determining the strength of insurers' dynamic risk-taking incentives.

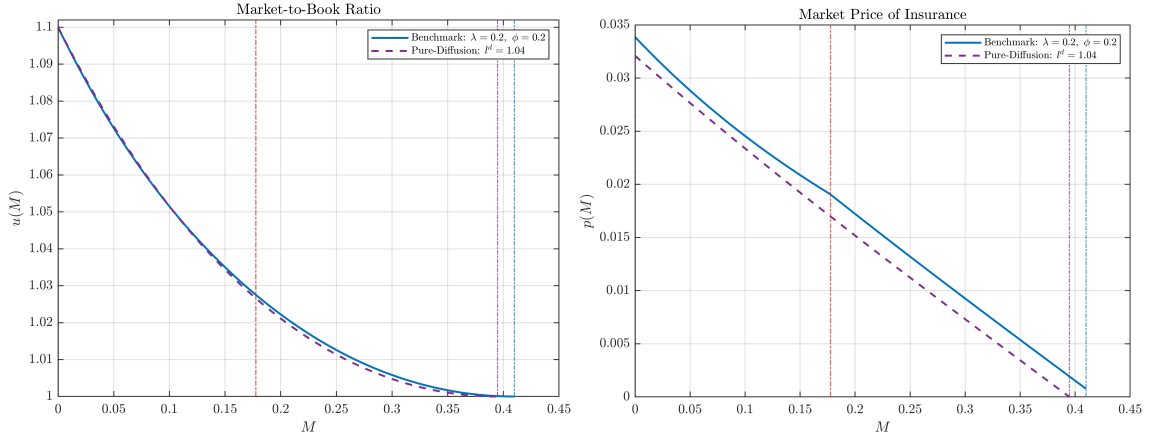


Figure 3.3: Benchmark Equilibrium under Infrequent Moderate Jumps: $(\lambda, \phi) = (0.2, 0.2)$.

As in the frequent-jump environment, the projection constraint is state dependent. The switching point is approximately $M_k = 0.178$: below this threshold, a jump moves the industry to the recapitalization boundary, whereas above it, the loss is absorbed within the internal financing region. The price schedule therefore retains a projection-induced kink.

Unlike the high-frequency case, however, the price remains monotonically decreasing over the entire capacity interval. A lower arrival frequency reduces the dynamic importance of repeated post-jump recapitalization and weakens the gamble-for-resurrection incentive. The conventional scarcity effect consequently remains dominant: as aggregate capacity increases, the shadow cost

of capital declines and equilibrium insurance prices fall. This comparison shows that projection switching is sufficient to create a change in the slope of the pricing schedule, but it is not sufficient to generate a reversal of that slope. Non-monotonicity requires the recapitalization-induced risk-taking incentive to become sufficiently strong relative to the scarcity effect.

Taken together, the three environments demonstrate that jump risk does not imply a unique equilibrium pricing pattern. Jump severity determines the extent of the projection-active region and whether the projection operator switches between financing regimes. Jump frequency affects the dynamic importance of the recapitalization option and hence the strength of the gamble-for-resurrection incentive. Their interaction determines whether the equilibrium price schedule is smooth, kinked but monotone, or kinked and non-monotone. A formal analytical characterization of the sources of non-smoothness and non-monotonicity is provided in [Appendix B](#).

These properties are economically important. A kink identifies a threshold at which the industry’s post-loss financing regime changes discretely, while non-monotonicity implies that market prices need not provide a one-to-one signal of aggregate capitalization. In the frequent-jump regime, where the price-capacity relationship becomes non-monotone, the same observed insurance price may be consistent with multiple capacity states. Consequently, insurers, regulators, and other market participants should not interpret price movements solely as actuarial responses to expected losses; they may also reflect state-dependent recapitalization constraints and risk-taking incentives arising from insurers’ balance sheets.

4 Robust Equilibrium with Model Uncertainty

4.1 Equilibrium Characterization

We now extend the benchmark equilibrium by introducing insurers’ concerns about model uncertainty in the catastrophe loss process. Rather than assuming that insurers fully trust the reference model, we allow them to consider alternative probability measures that are statistically close to the reference model but imply more adverse future outcomes. Under robust preferences, insurers jointly determine their underwriting strategy and the worst-case probability distortions. The detailed derivation of the HJBI equation, the optimal controls, and the associated boundary conditions is provided in [Appendix A.3](#). Combining these optimal decisions with the market-clearing

condition yields a system of equations characterizing the robust equilibrium.

Proposition 3 (Robust Equilibrium). *In the presence of model uncertainty, suppose that the following system of equations admits a solution for $p(\cdot)$ and $u(\cdot)$ on $[\underline{M}, \overline{M}]$:*

$$\left\{ \begin{array}{l} D(p(M)) = Mf(M), \\ ru(M) = D(p(M)) \left[\lambda\phi + (l + \lambda\phi)p(M) \right] u'(M) + \frac{1}{2} D(p(M))^2 \eta^2 u''(M) \\ \quad + \lambda \left[u(\Pi(M - D(p(M))\phi)) - u(M) \right] - \frac{\theta}{2} (h(M)^2 + \lambda g(M)^2) \\ \quad + f(M) \left\{ \left[\lambda\phi + (l + \lambda\phi)p(M) \right] u(M) + D(p(M)) \eta^2 u'(M) - \lambda\phi u(\Pi(M - D(p(M))\phi)) \right\}, \\ f(M) = \theta \left[\eta^2 u(M)^2 + \lambda\phi^2 u(\Pi(M - D(p(M))\phi))^2 \right]^{-1} \left\{ \left[\lambda\phi + (l + \lambda\phi)p(M) \right] u(M) \right. \\ \quad + D(p(M)) \eta^2 u'(M) \left(1 - \frac{u(M)}{\theta} \right) \\ \quad \left. + \lambda\phi u(\Pi(M - D(p(M))\phi)) \left(\frac{u(\Pi(M - D(p(M))\phi)) - u(M)}{\theta} - 1 \right) \right\}, \\ h(M) = -\frac{1}{\theta} \left[f(M)u(M) + D(p(M))u'(M) \right] \eta, \\ g(M) = \frac{1}{\theta} \left[(1 - f(M)\phi)u(\Pi(M - D(p(M))\phi)) - u(M) \right], \end{array} \right. \quad (4.1)$$

subject to the demand function $D(p(M)) = 1 - \frac{l+\lambda\phi}{\alpha(\eta^2+\lambda\phi^2)}p(M)$, the projection operator $\Pi(x) = \min \{ \overline{M}, \max \{ \underline{M}, x \} \}$, and the boundary conditions $u(\underline{M}) = 1 + \gamma$, $u(\overline{M}) = 1$, $u'(\underline{M}) = u'(\overline{M}) = 0$, together with the requirements in Assumption 1. Then, a stationary Markovian equilibrium exists:

(1) For $M \in (\underline{M}, \overline{M})$, the equilibrium insurance price is $p(M)$, the common market-to-book ratio is $u(M)$, and the worst-case density generators are $h(M)$ and $g(M)$. For an individual insurer with reserves m , the optimal underwriting scale is $x(m, M) = mf(M)$, and the shareholders' value is $v(m, M) = mu(M)$.

(2) For $M \geq \overline{M}$, insurers optimally distribute dividends to shareholders until aggregate reserves fall below $\overline{M}$. For $M \leq \underline{M}$, they raise external capital to restore reserves to $\underline{M}$.

The robust equilibrium introduces two additional endogenous objects, $h(M)$ and $g(M)$, which represent the optimal distortions to the diffusion and jump components of the catastrophe loss process, respectively. These density generators characterize the worst-case probability measure and capture the endogenous price of model uncertainty in equilibrium.

The introduction of model uncertainty modifies insurers' decisions through two main channels. First, ambiguity introduces an additional risk adjustment into equilibrium pricing. By evaluating underwriting decisions under adverse but plausible probability distortions, insurers require compensation not only for expected catastrophe losses and financial frictions, but also for the shadow cost associated with model uncertainty.

Second, ambiguity changes insurers' liquidity management incentives. Unlike the benchmark equilibrium, where the recapitalization boundary satisfies $\underline{M} = 0$, the robust equilibrium generally features a strictly positive lower boundary. This reflects the precautionary value of liquidity under model uncertainty: maintaining additional capital reserves helps insurers reduce exposure to unfavorable loss realizations and costly external financing.

Therefore, while the robust equilibrium retains the same barrier-type structure, ambiguity changes the endogenous capacity range and the pricing schedule, thereby affecting the dynamics of insurance markets. Conditional on the existence of the equilibrium, we next characterize insurers' expected underwriting profitability under the worst-case probability measure.

Proposition 4. *In equilibrium, insurers' expected underwriting profit under the optimally chosen measure $\mathbb{Q}^{h,g}$ is non-negative.*

The proof of this proposition is provided in Appendix A.4. Under the worst-case probability measure, insurers' expected underwriting profits reflect two components. The first component is the market price of insurance, $p(M)$, which compensates insurers for bearing catastrophe risk under the reference model. The second component arises from the probability distortions $(h(M), g(M))$, which represent the endogenous market price of model uncertainty associated with the diffusion and jump risks (Anderson et al., 2003). Together, these components determine the effective risk-adjusted compensation required for underwriting catastrophe exposure.

The proposition shows that even after accounting for the worst-case probability distortion, insurers earn non-negative expected underwriting profits in equilibrium. This result indicates that equilibrium insurance prices incorporate not only expected catastrophe losses, but also the endogenous costs associated with model uncertainty and financial capacity constraints.

4.2 Quantitative Analysis

We now examine how model uncertainty modifies the equilibrium outcomes characterized in the benchmark model. Using the same annual calibration as in Section 3.2, we numerically solve the robust equilibrium system in (4.1). The robustness parameter is set to $\theta = 20$, following Miao and Rivera (2016). A lower value of θ corresponds to stronger concerns about model uncertainty. Alternative values of θ are considered in Section 5.2 to examine the sensitivity of the results.

The objective of this section is to compare the robust equilibrium with the benchmark equilibrium under identical jump-risk environments. In particular, we examine how ambiguity concerns affect three key economic mechanisms: the pricing of jump risk, insurers' balance-sheet liquidity management, and the risk-taking incentives associated with the recapitalization option. For each jump-risk environment, we compare three equilibria: (i) the benchmark equilibrium without model uncertainty; (ii) the robust equilibrium with the same jump-risk process; and (iii) the robust equilibrium without jump risk. The third comparison isolates the effect of ambiguity when the loss process contains only continuous fluctuations.

Compared with the benchmark equilibrium, robustness concerns introduce additional endogenous adjustments through the worst-case density generators $h(M)$ and $g(M)$. These distortions modify insurers' assessment of future loss realizations and affect both underwriting decisions and capital management policies. As a result, robust equilibrium prices generally exceed their benchmark counterparts, reflecting compensation for uncertainty about the underlying loss distribution.

4.2.1 Severe Jump Losses

We first consider the severe jump-risk environment $(\lambda, \phi) = (0.2, 2)$, where jump losses are relatively infrequent but large in magnitude. Figure 4.1 compares the robust equilibrium with the benchmark equilibrium and the robust no-jump counterfactual.

The robust equilibrium generates substantially higher insurance prices than the benchmark equilibrium. This difference reflects the additional compensation required by insurers when they evaluate future losses under an adverse probability measure. In particular, ambiguity increases the effective cost of bearing large discontinuous losses and induces insurers to demand higher equilibrium loadings. Model uncertainty also changes insurers' capacity management decisions. The robust

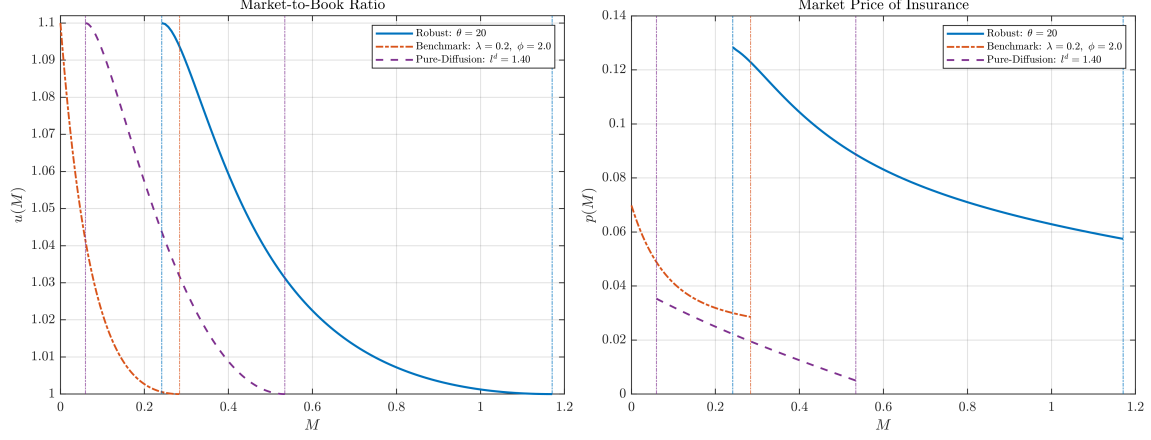


Figure 4.1: Robust Equilibrium under Severe Jump Losses: $(\lambda, \phi) = (0.2, 2)$.

equilibrium supports a larger capacity range before reaching the payout boundary, reflecting the precautionary value of capital under ambiguity. When insurers are uncertain about the true loss distribution, additional capital provides protection against unfavorable realizations and reduces exposure to future financing constraints.

Despite stronger risk adjustment, the qualitative price-capacity relationship remains monotone in this environment. Since severe jump losses make the projection constraint active over a large capacity region, ambiguity mainly raises the compensation required for jump exposure rather than altering the underlying scarcity-based pricing mechanism.

4.2.2 Frequent Moderate Jumps

We next consider the frequent moderate jump-risk environment $(\lambda, \phi) = (2, 0.2)$, which generates the strongest gamble-for-resurrection incentives and a non-monotone pricing relationship in the benchmark equilibrium. As shown in Figure 4.2, model uncertainty substantially weakens this mechanism. While the underlying projection-switching remains, model uncertainty reduces the magnitude of the associated kink and substantially attenuates the non-monotonic pricing pattern.

The difference arises from the interaction between ambiguity and insurers' risk-taking incentives. Under the benchmark model, limited liability and the possibility of recapitalization reduce the downside consequences of aggressive underwriting, creating a gamble-for-resurrection incentive. Under robust preferences, insurers evaluate future losses under more adverse probability measures and therefore assign greater importance to unfavorable jump realizations. This increases the effective cost

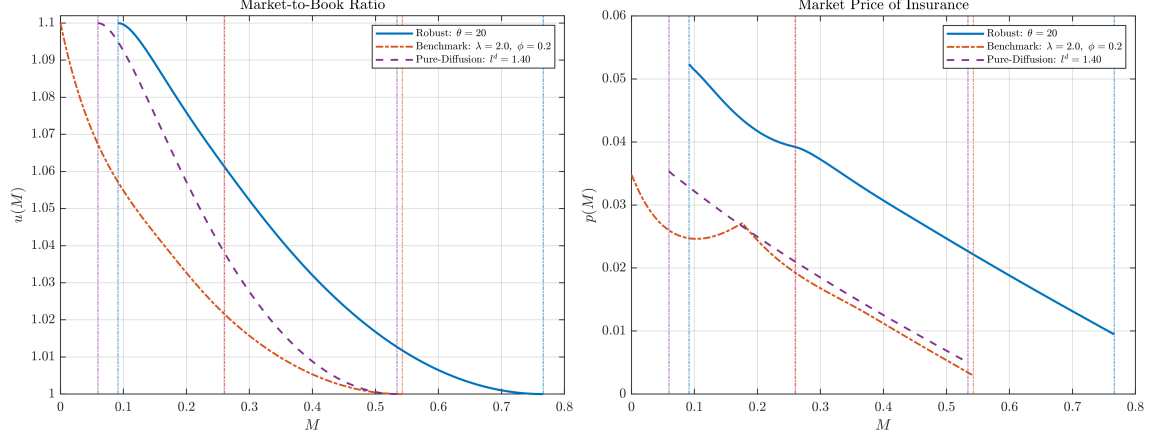


Figure 4.2: Robust Equilibrium under Frequent Moderate Jumps: $(\lambda, \phi) = (2, 0.2)$.

of aggressive underwriting and weakens the incentive to expand exposure in financially constrained states. Thus, model uncertainty does not eliminate the jump-induced financing mechanism itself. Instead, it changes the equilibrium transmission of this mechanism by reducing the strength of the risk-taking channel. A formal analysis of how ambiguity attenuates the pricing impact of projection switching is provided in Appendix C.

4.2.3 Infrequent Moderate Jumps

Finally, we examine the infrequent moderate jump-risk environment $(\lambda, \phi) = (0.2, 0.2)$. This case provides a comparison in which the benchmark equilibrium exhibits projection switching but does not generate a strong gamble-for-resurrection incentive. As shown in Figure 4.3, the robust equilibrium preserves the underlying financing transition while reducing the pricing impact of the projection-switching mechanism.

Compared with the frequent-jump environment, the effect of ambiguity is more subtle because the recapitalization option is less frequently exercised. Nevertheless, robustness concerns still increase equilibrium prices and encourage more conservative liquidity management. The comparison between the two moderate-jump environments highlights that ambiguity primarily operates through the endogenous assessment of downside risk: when jump losses arrive frequently, ambiguity substantially weakens excessive risk-taking incentives; when jump losses are less frequent, its main effect is to adjust risk compensation and reduce the pricing impact of transitions across financing regimes.

Taken together, these results show that model uncertainty does not simply amplify or eliminate

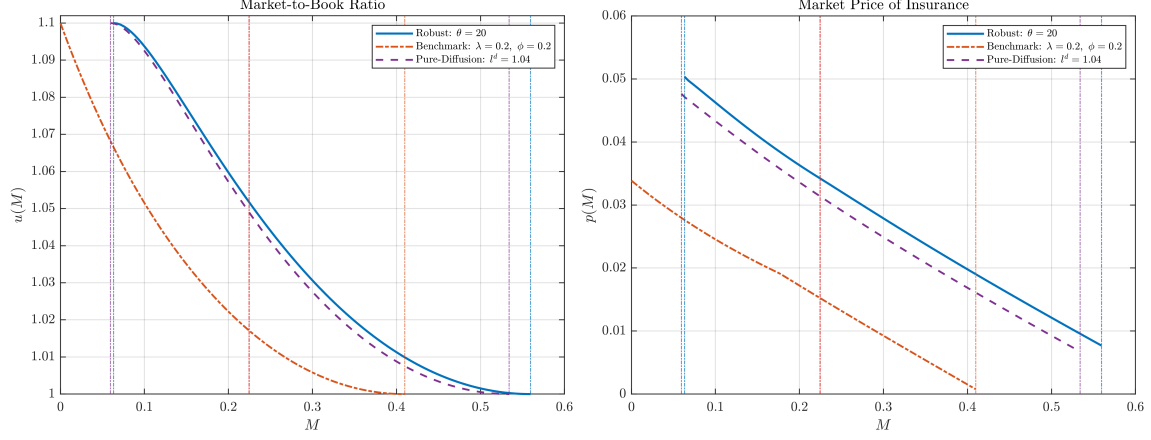


Figure 4.3: Robust Equilibrium under Infrequent Moderate Jumps: $(\lambda, \phi) = (0.2, 0.2)$.

the effects of jump risk. Instead, ambiguity reshapes the equilibrium transmission mechanism through which jump losses affect insurance prices and capacity dynamics. By increasing the effective cost of uncertain loss exposure, encouraging precautionary capital accumulation, and weakening gamble-for-resurrection incentives, robustness concerns attenuate pricing distortions and induce more conservative insurer behavior.

5 Implications for Catastrophe Insurance Markets

5.1 Market-Disciplined Calibration of Jump Risk

The previous sections consider different jump-risk environments to characterize the economic mechanisms through which discontinuous losses affect insurance pricing and capacity dynamics. In this section, we complement the mechanism analysis with a market-disciplined calibration of the jump-risk parameters. We do not aim to structurally estimate the equilibrium model. Instead, we discipline the magnitude of jump frequency and severity using evidence from catastrophe insurance markets and evaluate which theoretical pricing regime is most relevant in practice.

Maintaining the remaining parameter specifications used in Sections 3 and 4, we focus on the two jump-risk parameters: λ , which represents the arrival intensity of large-loss shocks, and ϕ , which captures the incremental loss associated with a jump event. We use publicly available global natural disaster loss data from the EM-DAT/Our World in Data database over the period 2000–2024. Although these data measure aggregate economic damages rather than insured losses, they provide a

transparent proxy for the frequency and severity of large-loss episodes that can materially affect insurers' balance sheets. We identify severe-loss years based on a pre-specified threshold relative to the normal annual loss scale. The detailed construction is provided in Appendix D.

The calibration procedure yields an estimated arrival frequency of $\hat{\lambda} = 0.12$, corresponding to approximately one major aggregate loss shock every eight years. For the severity parameter, severe-loss years generate total losses approximately 2.376 times the normal annual loss scale. Since ϕ represents the incremental jump component beyond the normal loss process, the corresponding jump severity is calibrated as $\hat{\phi} = 2.376 - 1 = 1.376$. This calibration is economically close to the severe jump-loss environment with $(\lambda, \phi) = (0.2, 2)$ considered in previous sections, where the regime is characterized by an always-active projection constraint: large loss shocks immediately exhaust internal capacity and trigger recapitalization. Although the recapitalization option generates a gamble-for-resurrection incentive, the relatively low jump frequency implies that this incentive is not strong enough to overturn the conventional capacity-scarcity effect.

To further quantify the role of jump risk, we compare the calibrated jump-risk economy with a matched pure-diffusion counterfactual that has the same expected loss, with $l^d = l + \lambda\phi = 1.16512$ and $\lambda^d\phi^d = 0$. As shown in Figure 5.1, jump risk substantially increases equilibrium insurance prices beyond the effect of higher expected losses alone. In the benchmark equilibrium, the maximum loading increases from 2.86% in the pure-diffusion economy to 4.91% in the jump-risk economy, while in the robust equilibrium it increases from 4.25% to 8.54%. This result highlights that discontinuous loss shocks introduce additional compensation for tail risk and balance-sheet uncertainty even after matching the expected loss level.

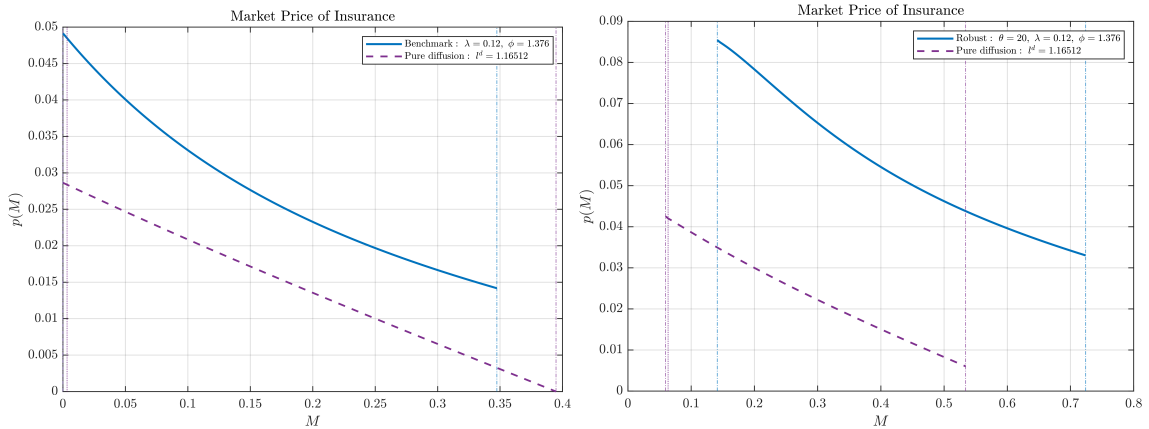


Figure 5.1: Equilibrium Insurance Prices under Jump Risk and Matched Pure Diffusion.

5.2 Sensitivity to Robustness Degree and Financing Friction

Based on the market-disciplined calibration, we next quantify how robustness concerns and financing frictions affect equilibrium outcomes.

5.2.1 Sensitivity to Robustness Degree

We first examine the quantitative implications of robustness concerns. As shown in Figure 5.2, stronger robustness concerns (i.e., lower θ) substantially expand the equilibrium capacity range and increase equilibrium insurance prices.

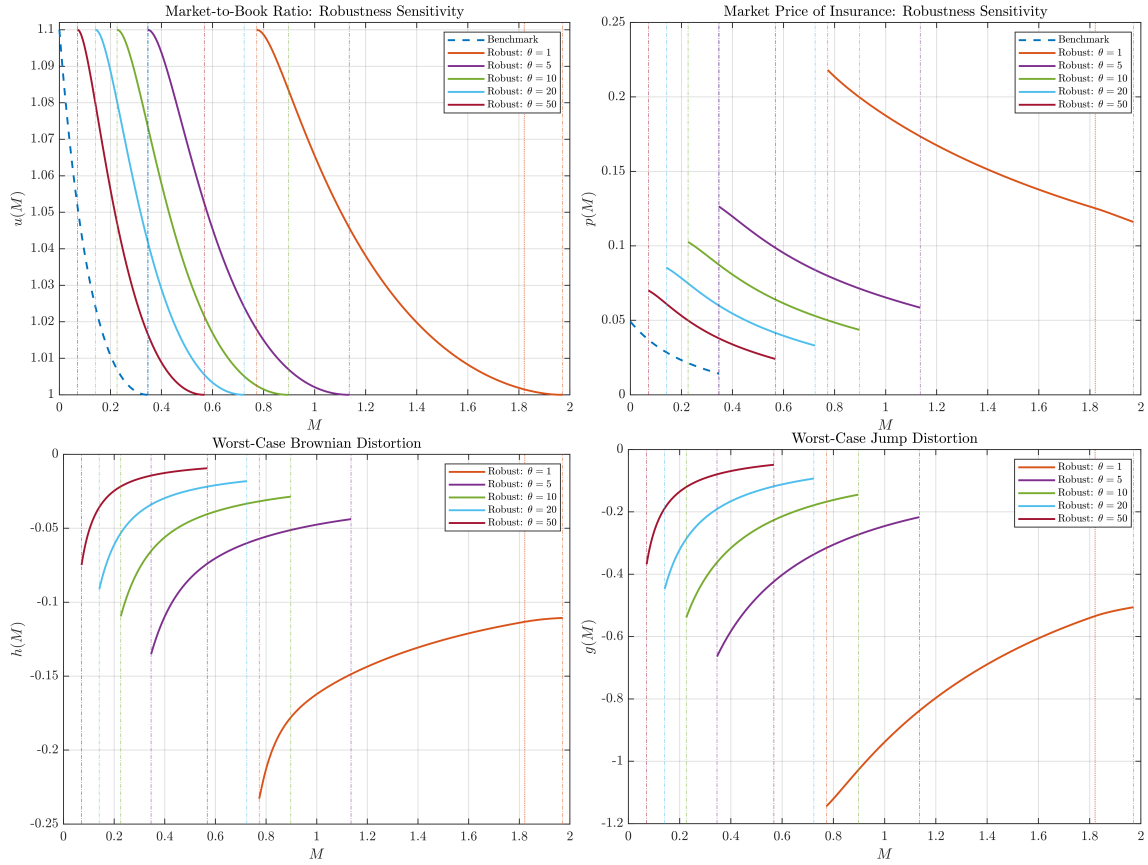


Figure 5.2: Equilibrium Outcomes for Different Robustness Degrees θ .

Quantitatively, the capacity interval expands substantially as robustness concerns become stronger. Relative to the benchmark equilibrium where the capacity range is $[0, 0.347]$, the robust equilibrium with $\theta = 20$ expands the range to $[0.141, 0.724]$, while the strongest robustness concern considered ($\theta = 1$) further expands it to $[0.773, 1.971]$. This reflects insurers' incentive to maintain larger capital buffers when they place greater weight on model uncertainty. In particular, insurers delay dividend

distributions and retain more capital before returning funds to shareholders.

Robustness concerns also generate a sizable increase in equilibrium insurance prices. In the benchmark equilibrium, the loading decreases from 4.91% at the lower capacity boundary to 1.42% at the upper capacity boundary. With stronger robustness concerns, the corresponding loadings become 8.54% and 3.30% for $\theta = 20$, and 21.81% and 11.60% for $\theta = 1$, respectively. Thus, robustness concerns introduce a substantial additional risk premium for bearing uncertain loss exposures. Insurers facing identical expected catastrophe losses may charge substantially different premiums depending on the degree of robustness concerns and the associated balance-sheet implications.

The lower panels of Figure 5.2 show that both the diffusion-risk distortion $h(M)$ and jump-risk distortion $g(M)$ become larger in magnitude as θ decreases. Under stronger robustness concerns, the worst-case density generators become more negative, reflecting a larger distortion toward adverse probability measures and a higher implicit cost of model uncertainty. This additional cost is ultimately reflected in equilibrium insurance prices, as insurers require greater compensation for bearing losses under a more pessimistic assessment of future risk realizations.

5.2.2 Sensitivity to Financing Friction

We next examine how external financing conditions affect equilibrium outcomes by varying the recapitalization wedge γ . Figure 5.3 reports the results for $\gamma \in \{0.05, 0.10, 0.20\}$.

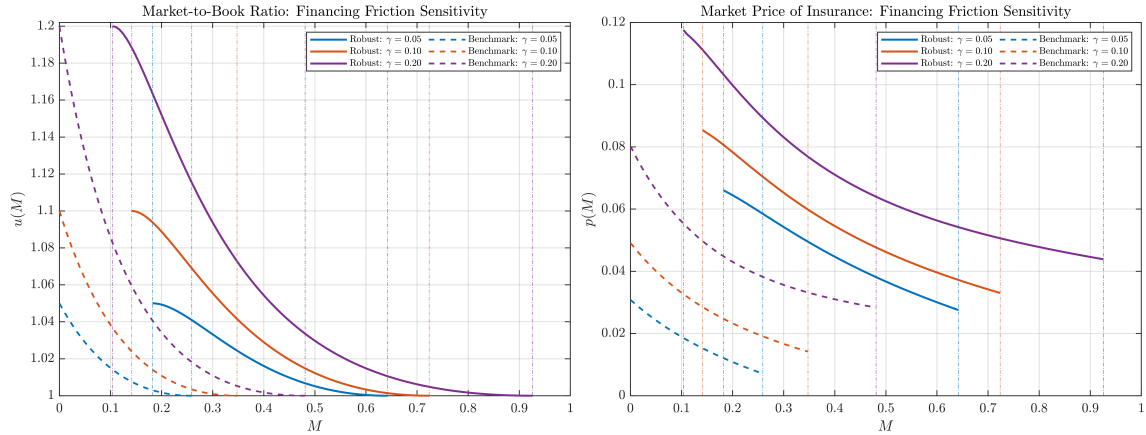


Figure 5.3: Equilibrium Outcomes for Different Financing Frictions γ .

Higher financing frictions lead to higher equilibrium prices and larger market-to-book ratios. When external capital becomes more costly, insurers require greater compensation for the cost of

rebuilding capacity after adverse losses. The quantitative effects are substantial. In the benchmark equilibrium, increasing γ from 0.05 to 0.20 raises the maximum insurance loading from 3.08% to 8.03%, while the upper capacity boundary increases from 0.259 to 0.481. In the robust equilibrium with $\theta = 20$, the effect is even more pronounced. Increasing γ from 0.05 to 0.20 raises the maximum insurance loading from 6.61% to 11.76%, while the upper capacity boundary increases from 0.642 to 0.926. At the same time, the lower capacity boundary decreases from 0.182 to 0.104, indicating that higher external financing costs make it more difficult for insurers to maintain capital buffers and recover after adverse shocks. Taken together, these sensitivity analyses demonstrate that robustness concerns and financing frictions materially affect catastrophe insurance markets.

5.3 Dynamic Patterns and Underwriting Cycles

We next examine the dynamic implications of calibrated jump risk for insurance market fluctuations and underwriting cycles. Following the framework developed in previous sections, aggregate capacity evolves as a reflected stochastic process within its admissible range, and equilibrium insurance prices adjust endogenously with the evolution of capacity. The expected duration of soft and hard market phases is computed using the corresponding infinitesimal generators. The technical details are provided in Appendix E.

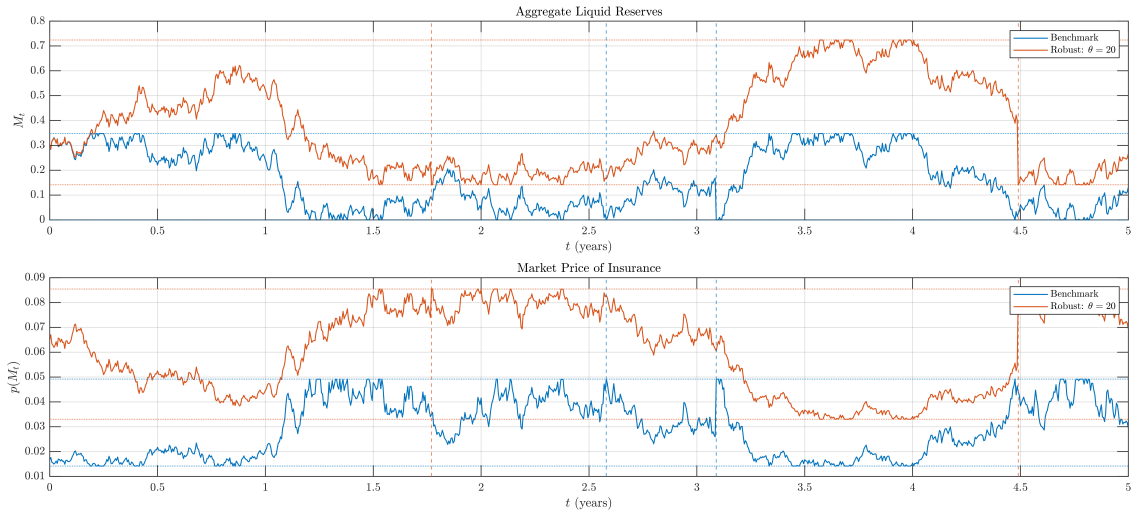


Figure 5.4: Dynamic Equilibrium Outcomes without and with Model Uncertainty.

Figure 5.4 presents simulated dynamic paths under the market-disciplined jump-risk calibration. The two economies are initialized with the same aggregate capacity ($M_0 = 0.3$) and are exposed to

identical Brownian shocks, while jump arrivals are simulated independently. In both equilibria, jump events generate sharp contractions in aggregate capacity and corresponding increases in insurance prices. Compared with the benchmark equilibrium, the robust equilibrium maintains higher capacity and persistently higher insurance prices.

To quantify the dynamic implications, we compute the expected duration of underwriting cycles. Table 1 reports the results under the calibrated jump-risk economy and its matched pure-diffusion counterfactual with the same expected loss. In the benchmark equilibrium, the expected cycle length is 3.54 years under jump risk, compared with 5.00 years in the pure-diffusion economy. The shorter cycle reflects the direct impact of jump shocks, which can rapidly reduce aggregate capacity and trigger transitions from soft to hard market conditions. In contrast, under the robust equilibrium, the expected cycle length increases substantially to 8.38 years under jump risk, compared with 8.52 years in the matched pure-diffusion economy. The comparison suggests that robustness concerns slow the adjustment of aggregate capacity and make underwriting cycles more persistent.

We further examine how the duration of underwriting cycles responds to robustness concerns and financing frictions. Stronger robustness concerns significantly lengthen the cycle by encouraging insurers to maintain larger capital buffers and slowing the adjustment of aggregate capacity. Quantitatively, the expected cycle length increases from 6.80 years when $\theta = 50$ to 16.92 years when $\theta = 1$. Similarly, higher financing frictions increase the duration of underwriting cycles because costly recapitalization delays capacity recovery after adverse shocks. In the robust equilibrium, increasing γ from 0.05 to 0.20 extends the expected cycle length from 5.98 to 11.40 years.

The estimated cycle durations are broadly comparable with the empirical literature on underwriting cycles, while also illustrating why empirical estimates may differ substantially across markets and time periods. Early empirical studies generally document underwriting cycles lasting several years, with estimates often falling in the range of approximately 6–10 years (Cummins and Outreville, 1987; Harrington et al., 2013). However, the empirical identification of a stable cycle length remains challenging, as estimates are sensitive to sample periods, filtering methods, and model specifications. In particular, Boyer et al. (2012) emphasize that statistical evidence for a persistent and predictable underwriting cycle is difficult to establish robustly.

Our model provides a potential explanation for this empirical variation. The sensitivity analysis shows that stronger robustness concerns and higher financing frictions can substantially extend the

Table 1: Numerically Solved Expected Underwriting Cycle Durations

Scenario	Benchmark			Robust ($\theta = 20$)		
	T_s	T_h	T_c	T_s	T_h	T_c
<i>Panel A: Calibrated Jump Risk and Matched Pure Diffusion</i>						
Jump risk	1.061	2.478	3.539	2.434	5.947	8.381
Matched diffusion	2.537	2.459	4.996	4.421	4.103	8.524
<i>Panel B: Robustness Sensitivity</i>						
$\theta = 1$	-	-	-	8.309	8.610	16.919
$\theta = 5$	-	-	-	3.909	6.531	10.440
$\theta = 10$	-	-	-	3.034	6.475	9.508
$\theta = 20$	-	-	-	2.434	5.947	8.381
$\theta = 50$	-	-	-	1.894	4.901	6.795
<i>Panel C: Financing Friction Sensitivity</i>						
$\gamma = 0.05$	0.644	1.243	1.888	1.702	4.277	5.979
$\gamma = 0.10$	1.061	2.478	3.539	2.434	5.947	8.381
$\gamma = 0.20$	1.775	4.948	6.723	3.984	7.415	11.398

Notes: T_s , T_h , and T_c denote the expected duration of the soft market phase, hard market phase, and the complete underwriting cycle, respectively. The baseline calibration uses $\lambda = 0.12$ and $\phi = 1.376$. The matched pure-diffusion economy sets $\lambda = 0$ and adjusts the loss drift to $\ell^d = \ell + \lambda\phi = 1.16512$. Robust equilibria use $\theta = 20$ unless otherwise specified.

duration of underwriting cycles. This suggests that the length of an observed cycle is not solely determined by the underlying loss process, but also depends on insurers' information environment and financing conditions. In particular, robustness concerns and financing frictions introduce additional persistence into capacity adjustments, implying that markets with different degrees of model uncertainty or external financing constraints may exhibit substantially different cycle lengths even under similar loss environments.

Moreover, underwriting cycles are inherently stochastic, so estimating a stable average cycle length requires observing a sufficiently large number of complete cycles. However, when financial frictions or robustness concerns generate longer cycles, historical insurance data may contain only a limited number of complete cycles, making precise statistical identification challenging. Therefore, the empirical difficulty in estimating underwriting cycle length may reflect not only methodological uncertainty, but also the economic heterogeneity and long-lived dynamics generated by insurers' balance-sheet conditions. This perspective helps reconcile the mixed empirical evidence on the existence and duration of underwriting cycles: the absence of a universal cycle length does not necessarily imply the absence of cyclical dynamics, but may reflect variation in the institutional environment faced by insurers.

6 Further Discussion

6.1 Extensions and Robustness of the Model Framework

Our baseline theoretical framework is intentionally designed to characterize the equilibrium mechanisms in a tractable environment. This section discusses two natural extensions. Additional extensions regarding loss severity, state-dependent financing conditions, and insurer heterogeneity are discussed in [Appendix F](#).

6.1.1 Alternative Demand Function

The baseline model follows [Luciano and Rochet \(2022\)](#) and adopts a linear demand function for tractability and analytical convenience. This specification allows the equilibrium conditions to be expressed in a relatively simple form and facilitates the numerical characterization of the equilibrium. A more general formulation could replace the linear demand function with an arbitrary downward-

sloping demand schedule. Under such an extension, the equilibrium conditions characterized in Propositions 1 and 3 would retain the same structure. The resulting equilibrium would generally require a numerical solution rather than the current tractable specification.

Importantly, the main economic forces identified in this paper do not rely on the linearity of the demand function. As long as insurance demand decreases with price, limited capacity generates a scarcity effect, while the recapitalization option creates an additional risk-taking incentive after large losses. A richer demand specification would therefore affect the quantitative magnitude of these effects but not the fundamental mechanisms studied in this paper.

6.1.2 Asset-Side Decisions

The baseline model focuses on insurers' liability-side decisions, including underwriting and pricing, in order to isolate how catastrophe-related jump losses affect insurance market equilibrium. A natural extension is to incorporate insurers' asset-side portfolio choices and investment-return shocks.

In the literature, [Luciano and Rochet \(2022\)](#) study insurers' asset-side portfolio choice and its implications for insurance cycles and asset prices, but do not explicitly incorporate jump risk or model uncertainty. More recently, [Chen et al. \(2026\)](#) develop a framework that jointly considers underwriting, investment, and liquidity management under model uncertainty and financial frictions. Extending the current framework to incorporate asset-side decisions would provide a more comprehensive characterization of insurers' risk management choices. However, jointly modeling investment, underwriting, and liquidity decisions substantially increases the complexity of the equilibrium system. The present paper therefore abstracts from asset-side behavior to isolate the role of catastrophe-related jump losses and recapitalization constraints.

The interaction between asset-side and liability-side risk-taking incentives provides an important direction for future research. While [Beyer \(2025\)](#) documents state-dependent and non-monotonic portfolio risk-taking among insurers during crisis periods, this paper identifies a related risk-taking channel arising from liability-side underwriting decisions after large loss shocks. A unified framework combining these two channels may provide further insights into insurers' dynamic risk-taking behavior.

6.2 Empirical Relevance and Testable Predictions

The model generates several empirically testable predictions regarding catastrophe insurance pricing, capacity dynamics, and insurer behavior. Table 2 summarizes the main predictions, their connections with existing empirical findings, and potential directions for future research.

Table 2: Empirical Predictions and Related Evidence

Model Prediction	Related Empirical Evidence	Potential Empirical Test
<i>Jump risk affects insurance prices beyond expected losses.</i>	Catastrophe insurance prices often exceed actuarial expected losses, and capital supply conditions play an important role in catastrophe risk pricing (Froot and O’Connell, 1999; Froot, 1999; Eling et al., 2023, 2025).	Examine whether catastrophe severity and frequency predict premiums after controlling for expected losses and standard actuarial risk measures.
<i>Aggregate capacity and financing conditions affect insurance pricing.</i>	Capacity constraints and external financing conditions have been shown to influence insurance cycles and premium dynamics (Gron, 1994a; Gron and Lucas, 1998).	Study the interaction between catastrophe exposure, insurer capitalization, external financing conditions, and premium adjustments.
<i>Robustness concerns generate an uncertainty premium.</i>	Existing studies document risk premia in insurance markets, but the role of model uncertainty in catastrophe pricing remains less explored.	Investigate whether measures of catastrophe model uncertainty, regulatory uncertainty, or information quality explain premium differences beyond expected losses.
<i>Robustness concerns and financing frictions affect underwriting cycles.</i>	Empirical studies document multi-year underwriting cycles (Cummins and Outreville, 1987; Harrington et al., 2013), while the estimated cycle length varies across methodologies (Boyer et al., 2012).	Examine whether differences in insurers’ uncertainty assessments and financing conditions explain cross-market variation in cycle duration.
<i>Catastrophe losses may generate state-dependent risk-taking incentives.</i>	Recent evidence documents nonlinear insurer risk-taking behavior during crisis periods, mainly focusing on asset-side portfolio decisions (Beyer, 2025).	Test whether insurers with depleted capitalization adjust underwriting exposure and pricing differently during recovery periods.

6.3 Managerial and Policy Implications

Although the framework is stylized, it provides implications for insurers, consumers, and regulators in catastrophe-exposed markets. The main trade-off highlighted by the model is that robustness concerns and stronger financial discipline can improve insurer resilience and market stability, but may also increase insurance costs and reduce affordability.

6.3.1 Implications for Insurers

For insurers, the model provides a framework for determining appropriate premium loadings in catastrophe-exposed lines. Rather than pricing solely based on expected losses, insurers should incorporate the additional compensation required for discontinuous jump risk, uncertainty about the loss distribution, and the cost of recapitalization. In this sense, capital and liquidity management are not merely background constraints, but integral components of catastrophe underwriting decisions.

The quantitative analysis illustrates the importance of these considerations. Under the calibrated jump-risk environment, the equilibrium loading is substantially higher than in a matched pure-diffusion economy with the same expected loss, indicating that catastrophe jump risk generates an additional pricing premium beyond actuarial losses. This effect is further amplified by robustness concerns and financing frictions, as insurers require greater compensation when future loss realizations are more uncertain or when rebuilding capacity becomes more costly.

This perspective complements the growing supply-side view of insurance markets, which emphasizes the role of insurers' balance-sheet conditions and financing frictions in shaping prices and quantities (Koijen and Yogo, 2021, 2022; Ge, 2022). The model also highlights the importance of distinguishing prudent risk adjustment from excessive risk-taking: while robustness concerns promote more conservative capital management, limited recognition of tail risk and recapitalization constraints may generate liability-side gamble-for-resurrection incentives.

6.3.2 Implications for Consumers and Protection Gaps

For consumers, the model highlights a fundamental trade-off between insurance affordability and insurer financial resilience. Robust pricing and stronger capital discipline improve insurers' ability to absorb catastrophic losses, but they also increase equilibrium premiums. Higher prices may reduce insurance demand among households with limited risk-bearing capacity and potentially widen protection gaps in catastrophe-exposed regions.

However, the model does not imply that lower premiums necessarily improve social welfare. Insufficient pricing of catastrophe risk may undermine insurer solvency and reduce the reliability of insurance protection after severe events. Therefore, the effectiveness of catastrophe insurance markets depends not only on premium affordability but also on the sustainability of risk transfer

mechanisms. A complete analysis of protection gaps would require incorporating heterogeneous consumers, demand-side constraints, and the social value of insurance coverage, which are beyond the scope of the current framework.

6.3.3 Implications for Regulators and Welfare Trade-offs

For regulators, the model highlights the trade-off between financial stability and insurance accessibility. Policies that encourage stronger capitalization and more conservative risk assessment can reduce insurer distress and improve market resilience. However, excessive capital requirements or higher external financing costs may increase premiums and limit insurance availability.

Conversely, policies that facilitate recapitalization, such as reinsurance, catastrophe bonds, contingent capital, or public-private risk-sharing mechanisms, may improve market capacity by reducing post-event financing constraints. The catastrophe insurance literature emphasizes that the availability of risk-bearing capital is central to market capacity and pricing ([Jaffee and Russell, 1997](#); [Froot, 2001](#); [Harrington and Niehaus, 2003](#)).

The model therefore suggests that effective policy design requires balancing two objectives: maintaining insurer solvency and preserving affordable access to catastrophe protection. Robust pricing can improve financial stability by internalizing uncertainty and capital constraints, but its welfare benefits depend on whether the resulting increase in premiums is outweighed by the improvement in the reliability of insurance protection.

7 Conclusion

This paper develops a dynamic competitive equilibrium framework of insurance capacity and pricing to study how catastrophe jump risk interacts with financial constraints and model uncertainty. Building on the continuous-risk models of [Henriet et al. \(2016\)](#) and [Pang \(2026\)](#), we introduce discontinuous loss shocks into a competitive insurance market with endogenous capital management. A central contribution of the paper is to identify a liability-side gamble-for-resurrection mechanism arising from the interaction between jump losses, limited liability, and recapitalization options. While jump risk does not imply a unique pricing pattern, it can fundamentally reshape the capacity-price relationship by changing the relative strength of the conventional scarcity effect and the

recapitalization incentive. Introducing model uncertainty further modifies this mechanism: robustness concerns increase the compensation required for uncertain loss exposure, encourage precautionary capital accumulation, and attenuate excessive risk-taking incentives after large losses.

Using a market-disciplined calibration of catastrophe jump risk, we show that discontinuous losses have quantitatively important pricing implications even when expected losses are held fixed. The model also generates endogenous underwriting cycles whose duration depends significantly on robustness concerns and financing frictions. These results highlight that catastrophe insurance pricing reflects not only actuarial risk, but also insurers' balance-sheet conditions, access to external capital, and uncertainty about future loss realizations.

More broadly, this paper provides a framework for understanding supply-side determinants of catastrophe insurance markets and the trade-off between financial stability and insurance affordability. Future research could extend the framework by incorporating heterogeneous insurers, asset-side portfolio decisions, reinsurance and capital-market instruments, or state-dependent robustness concerns to further study insurer resilience and risk-sharing in catastrophe-exposed markets.

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A Technical Derivations and Proofs

A.1 Derivation of Benchmark Equilibrium Conditions

In this section, we derive insurers' optimal decisions and the corresponding market equilibrium conditions in the absence of model uncertainty. Based on (2.6), an insurer's value function $v(m, M)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} rv(m, M) = \sup_{x \geq 0, d\delta \geq 0, di \geq 0} & \left\{ x \left[\lambda\phi + (l + \lambda\phi)p \right] v_m + D(p) \left[\lambda\phi + (l + \lambda\phi)p \right] v_M + \frac{1}{2} x^2 \eta^2 v_{mm} \right. \\ & + \frac{1}{2} D(p)^2 \eta^2 v_{MM} + x D(p) \eta^2 v_{mM} + \lambda \left[v(m - x\phi, M - D(p)\phi) - v(m, M) \right] \\ & \left. + d\delta(1 - v_m) + di(v_m - 1 - \gamma) \right\}. \end{aligned} \quad (\text{A.1})$$

A.1.1 Internal Financing Region

We begin by analyzing the internal financing region, where the optimal dividend and recapitalization policies are $d\delta = 0$ and $di = 0$. Given the assumption $v(m, \cdot) = mu(\cdot)$, we have $v_{mm} = 0$ and $v(m - x\phi, M - D(p)\phi) = (m - x\phi)u(M - D(p)\phi)$. Substituting them into (A.1), the remaining problem reduces to a linear optimization with respect to the underwriting scale x . An interior solution exists if and only if:

$$\left[\lambda\phi + (l + \lambda\phi)p \right] u(M) + D(p) \eta^2 u'(M) - \lambda\phi u(M - D(p)\phi) = 0. \quad (\text{A.2})$$

We exclude the possibility of a corner solution $x = 0$, which would arise if the left-hand side of (A.2) were negative. In such a case, the market-clearing condition would imply $D(p) = 0$, and the negative sign of the left-hand side would further suggest $p < 0$, contradicting the demand function (2.3), which requires $D(p) > 1$ whenever $p < 0$.

Substituting (A.2) back into the HJB equation, the expression simplifies to:

$$ru(M) = D(p) \left[\lambda\phi + (l + \lambda\phi)p \right] u'(M) + \frac{1}{2} D(p)^2 \eta^2 u''(M) + \lambda \left[u(M - D(p)\phi) - u(M) \right]. \quad (\text{A.3})$$

Together with (A.2), this system jointly determines the equilibrium functions $p(\cdot)$ and $u(\cdot)$. Notably, setting $\lambda\phi = 0$ reduces these equations to those in [Henriet et al. \(2016\)](#), indicating that our framework

nesting their continuous-risk model as a special case.

However, the introduction of the jump term fundamentally alters the structure of the system. In particular, when a jump occurs, the term $M - D(p)\phi$ may fall outside the domain of definition of $u(\cdot)$, which is originally restricted to $[\underline{M}, \overline{M}]$. When the aggregate capacity falls below $\underline{M}$, insurers are assumed to raise external capital to restore reserves back to $\underline{M}$. To handle such cases formally, we introduce a projection operator $\Pi(x) \triangleq \min\{\overline{M}, \max\{\underline{M}, x\}\}$. The term $u(M - D(p)\phi)$ in (A.2) and (A.3) should be replaced by $u(\Pi(M - D(p)\phi))$.

A.1.2 External Financing and Payout Regions

Next, we analyze the optimal dividend and recapitalization policies. Based on the HJB equation (A.1), a necessary condition for the existence of a solution is:

$$1 - v_m \leq 0, \quad v_m - 1 - \gamma \leq 0,$$

which implies that $1 \leq u(M) \leq 1 + \gamma$. Moreover, the optimal dividend policy satisfies $d\delta > 0$ only if $u(M) = 1$, whereas the recapitalization policy satisfies $di > 0$ only if $u(M) = 1 + \gamma$.

Following the definition of a barrier-type equilibrium, we impose the value-matching conditions:

$$u(\underline{M}) = 1 + \gamma, \quad u(\overline{M}) = 1.$$

These conditions capture the indifference of insurers at the boundaries: at $\overline{M}$, the insurer is indifferent between retaining or distributing one additional unit of reserves, while at $\underline{M}$, the marginal value of cash equals the marginal cost of recapitalization.

We also adopt the industry-level no-arbitrage condition proposed in [Henriet et al. \(2016\)](#) and [Luciano and Rochet \(2022\)](#):

$$V'(\underline{M}) = 1 + \gamma, \quad V'(\overline{M}) = 1,$$

where $V(M) = Mu(M)$ denotes the market value of the entire insurance sector. Economically, these conditions require that the marginal change in the total market value induced by a dividend payout or recapitalization equals the marginal flow of funds withdrawn from or injected into the industry by shareholders. At the upper boundary, the condition implies $u'(\overline{M}) = 0$ (since $\overline{M} > 0$). At the lower

boundary, it implies either $\underline{M} = 0$ or $u'(\underline{M}) = 0$.

We illustrate that the condition $u'(\underline{M}) = 0$ cannot generate a valid equilibrium in this setting. Suppose, to the contrary, that it holds. Then, from (A.2), we obtain $p(\underline{M}) = 0$ and $D(p(\underline{M})) = 1$. Substituting these into equation (A.3) yields $u''(\underline{M}) = \frac{2}{\eta^2}r(1 + \gamma) > 0$. This result contradicts the second condition in Assumption 1, which requires $u'(M) \leq 0$ for all $M \in [\underline{M}, \overline{M}]$. Indeed,

$$u''(\underline{M}) = \lim_{M \rightarrow \underline{M}^+} \frac{u'(M) - u'(\underline{M})}{M - \underline{M}} = \lim_{M \rightarrow \underline{M}^+} \frac{u'(M)}{M - \underline{M}} \leq 0.$$

which is inconsistent with the positive value implied by (A.3). Therefore, we adopt the condition $\underline{M} = 0$, implying that the insurance industry resorts to recapitalization only when its aggregate reserves are completely depleted.

A.2 Proof of Proposition 2

From the first equation in the equilibrium system, we have:

$$\begin{aligned} 0 &= (l + \lambda\phi)p(M)u(M) + D(p(M))\eta^2 u'(M) + \lambda\phi \left[u(M) - u(\Pi(M - D(p(M))\phi)) \right] \\ &\leq (l + \lambda\phi)p(M)u(M), \end{aligned}$$

where the inequality follows from the monotonicity condition for $u(\cdot)$. Therefore, $p(M) \geq 0$, implying that the expected underwriting profit per unit of insurance, $(l + \lambda\phi)p(M)$, is non-negative. $\square$

A.3 Derivation of Robust Equilibrium Conditions

In this section, we derive insurers' optimal decisions and the corresponding market equilibrium conditions in the presence of model uncertainty. Based on (2.7), an insurer's value function $v(m, M)$

satisfies the following Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation:

$$\begin{aligned}
rv(m, M) = & \sup_{x \geq 0, d\delta \geq 0, di \geq 0} \inf_{(h, g) \in \mathcal{B}} \left\{ \frac{1}{2} \Theta(m) (h^2 + \lambda g^2) + x [\lambda \phi + (l + \lambda \phi)p + \eta h] v_m \right. \\
& + D(p) [\lambda \phi + (l + \lambda \phi)p + \eta h] v_M + \frac{1}{2} x^2 \eta^2 v_{mm} + \frac{1}{2} D(p)^2 \eta^2 v_{MM} + x D(p) \eta^2 v_{mM} \\
& \left. + \lambda(1 - g) [v(m - x\phi, M - D(p)\phi) - v(m, M)] + d\delta(1 - v_m) + di(v_m - 1 - \gamma) \right\}.
\end{aligned} \tag{A.4}$$

A.3.1 Internal Financing Region

Similarly, we begin by analyzing the internal financing region. For $m = 0$, it follows that $\Theta(m) = 0$. Given the functional form $v(m, M) = mu(M)$, the minimization over (h, g) admits an interior solution only if $x = 0$. In this case, the HJBI equation holds trivially, as both sides equal zero. It implies that an individual insurer with zero liquid reserves will abstain from underwriting activity.

For $m > 0$, the minimization over h yields the interior solution:

$$h = -\frac{x\eta v_m + D(p)\eta v_M}{\Theta(m)},$$

and the minimization over $g \leq 1$ is attained at:

$$g = \min \left\{ \frac{v(m - x\phi, M - D(p)\phi) - v(m, M)}{\Theta(m)}, 1 \right\}.$$

In fact, we can safely restrict to the case $g \leq 0$, implying that the constraint $g \leq 1$ is non-binding in equilibrium. To see this, note that:

$$\begin{aligned}
v(m - x\phi, M - D(p)\phi) &= m \left(1 - \frac{x}{m} \phi \right) u(M - D(p)\phi) = \frac{m}{M} (M - D(p)\phi) u(M - D(p)\phi) \\
&= \frac{m}{M} V(M - D(p)\phi) \leq \frac{m}{M} V(M) = v(m, M),
\end{aligned}$$

where we use the equilibrium allocation condition $x = \frac{m}{M} D(p)$, which follows from the homogeneity of insurers and the market-clearing condition. The inequality follows from the pre-assumption that the aggregate value of the insurance sector $V(\cdot)$ is monotonically increasing, which is economically

reasonable and can be verified in the numerical analysis.

Substituting the minimizing distortions into the HJBI equation, the first-order condition with respect to x can be obtained directly from the envelope theorem. In particular, the terms involving $\partial h/\partial x$ and $\partial g/\partial x$ vanish at the minimizing distortions. Therefore,

$$0 = \left[\lambda\phi + (l + \lambda\phi)p + \eta h \right] v_m + x\eta^2 v_{mm} + D(p)\eta^2 v_{mM} - \lambda\phi(1 - g)v_m(m - x\phi, M - D(p)\phi).$$

The optimal underwriting scale is then derived as:

$$\begin{aligned} x(m, M) &= m\theta \left[\eta^2 u(M)^2 + \lambda\phi^2 u(M - D(p)\phi)^2 \right]^{-1} \left\{ \left[\lambda\phi + (l + \lambda\phi)p \right] u(M) \right. \\ &\quad \left. + D(p)\eta^2 u'(M) \left(1 - \frac{u(M)}{\theta} \right) + \lambda\phi u(M - D(p)\phi) \left(\frac{u(M - D(p)\phi) - u(M)}{\theta} - 1 \right) \right\} \\ &\triangleq mf(M), \end{aligned} \tag{A.5}$$

which is optimal when it is non-negative. Otherwise, the optimal scale is restricted to $x(m, M) = 0$, due to the non-negativity constraint.

In fact, a necessary condition for equilibrium existence is that $f(M) > 0$ for any admissible M . Otherwise, there exists an M such that $x(m, M) = 0$ for all $m > 0$. Combined with $x(0, M) = 0$, the market-clearing condition implies $D(p(M)) = 0$. Then, the optimal density generators are $h(M) = 0$ and $g(M) = 0$. Substituting them back into the HJBI equation leads to $rv(m, M) = 0$, which contradicts the assumption that $m > 0$ and $u(M) > 0$.

Based on (A.5), the HJBI equation can be expressed as:

$$\begin{aligned} ru(M) &= D(p) \left[\lambda\phi + (l + \lambda\phi)p \right] u'(M) + \frac{1}{2} D(p)^2 \eta^2 u''(M) + \lambda \left[u(M - D(p)\phi) - u(M) \right] \\ &\quad - \frac{\theta}{2} (h^2 + \lambda g^2) + f(M) \left\{ \left[\lambda\phi + (l + \lambda\phi)p \right] u(M) + D(p)\eta^2 u'(M) - \lambda\phi u(M - D(p)\phi) \right\}, \end{aligned} \tag{A.6}$$

where the density generators are given by:

$$h(M) = -\frac{1}{\theta} \left[f(M)u(M) + D(p)u'(M) \right] \eta, \quad g(M) = \frac{1}{\theta} \left[(1 - f(M)\phi)u(M - D(p)\phi) - u(M) \right].$$

Associated with the market-clearing condition:

$$D(p(M)) = Mf(M), \quad (\text{A.7})$$

where $D(p(M))$ equals the sum of $x(m, M)$, and the multiplier M on the right side is the sum of m across insurers, the equilibrium functions $p(\cdot)$ and $u(\cdot)$ can be jointly determined. When setting $\lambda\phi = 0$, the equation system reduces to that in Pang (2026). Similar to the setting in Section 3, the argument $M - D(p)\phi$ may also fall outside the domain of definition of $u(\cdot)$. Therefore, we replace the term $u(M - D(p)\phi)$ with $u(\Pi(M - D(p)\phi))$.

A.3.2 External Financing and Payout Regions

Similar to the analysis in Subsection A.1.2, we impose the following boundary conditions:

$$u(\underline{M}) = 1 + \gamma, \quad u(\overline{M}) = 1, \quad \underline{M}u'(\underline{M}) = 0, \quad u'(\overline{M}) = 0,$$

which ensure that, at both the payout and recapitalization boundaries, the marginal change in industry value corresponds precisely to the marginal change in total liquidity, either distributed to or injected by shareholders. We show that the contingent condition $\underline{M} = 0$ cannot generally hold once model uncertainty is incorporated. Suppose, for contradiction, that $\underline{M} = 0$ and there exist equilibrium functions $p(\cdot)$ and $u(\cdot)$. Then, the market-clearing condition (A.7) implies $D(p(0)) = 0$, which leads to the implied equilibrium price $p(0) = \frac{\alpha(\eta^2 + \lambda\phi^2)}{l + \lambda\phi}$. Substituting them back into the equation (A.6) yields:

$$r(1 + \gamma) = \frac{\theta}{2}\alpha^2(\eta^2 + \lambda\phi^2),$$

which holds only for a knife-edge set of parameter values. Therefore, except for such special parameter configurations, the robust equilibrium must have a strictly positive recapitalization boundary. We hence focus on the generic case $\underline{M} > 0$, for which the lower-boundary condition is $u'(\underline{M}) = 0$.

A.4 Proof of Proposition 4

Under the optimal measure, the jump intensity is $\lambda^g(M) \triangleq \lambda[1 - g(M)]$. Accordingly, the expected underwriting profit per unit of insurance is

$$\begin{aligned} & (l + \lambda\phi)(1 + p(M)) - \left(l - \eta h(M) + \lambda^g(M)\phi \right) \\ &= -D(p(M))\eta^2 \frac{u'(M)}{u(M)} + \lambda^g(M)\phi \frac{u(\Pi(M - D(p(M))\phi)) - u(M)}{u(M)}. \end{aligned}$$

Since $u'(M) \leq 0$, $\Pi(M - D(p(M))\phi) \leq M$, and $u(\cdot)$ is non-increasing, both terms on the right-hand side are non-negative. Therefore, the expected underwriting profit is non-negative. $\square$

B Pricing Regimes in Benchmark Equilibrium

This appendix provides an analytical interpretation of the pricing regimes identified in the numerical analysis. In particular, we characterize how the projection mechanism generates non-smooth pricing and how the interaction between capacity scarcity and the recapitalization option can generate non-monotonic price-capacity relationships.

B.1 Projection Switching and Non-smooth Pricing

Recall that the post-jump capacity is given by $M^J(M) \triangleq M - \phi D(p(M))$, and the projection operator maps the post-jump state back into the admissible capacity region. A projection-switching point arises when the post-jump capacity crosses the recapitalization boundary.

Proposition 5. *Suppose that there exists an interior capacity level $M_k \in (\underline{M}, \overline{M})$ satisfying $M^J(M_k) = \underline{M}$, and that the crossing is non-degenerate, i.e., $M^{J'}(M_k) \neq 0$. If $u'(\underline{M}) \neq 0$, then the post-jump continuation value $M \mapsto u(\Pi(M^J(M)))$ is continuous but generally not differentiable at M_k . Consequently, unless the effect is offset by a knife-edge cancellation in the remaining equilibrium conditions, the equilibrium price function exhibits a kink at M_k .*

The proposition shows that the kink observed in the numerical analysis is not a numerical artifact introduced by the projection operator. Instead, it reflects an endogenous transition between two economically distinct post-loss financing regimes. Below the switching point, a jump loss exhausts

internal capacity and forces insurers to rely on external recapitalization. Above the switching point, insurers absorb the loss within their existing balance sheets, and the post-jump continuation value varies with remaining capacity.

B.2 Recapitalization Option and Non-monotonic Pricing

We next characterize the mechanism generating non-monotonic pricing. Starting from the first equilibrium condition in Proposition 1, the equilibrium price can be decomposed as $p(M) = \mathcal{S}(M) + \mathcal{G}(M)$, where

$$\mathcal{S}(M) \triangleq -\frac{D(p(M))\eta^2 u'(M)}{(l + \lambda\phi)u(M)}, \quad \mathcal{G}(M) \triangleq \frac{\lambda\phi [u(\Pi(M^J(M))) - u(M)]}{(l + \lambda\phi)u(M)}.$$

The first component $\mathcal{S}(M)$ captures the conventional scarcity value of underwriting capacity. Since additional capacity reduces the marginal value of insurer capital, this force typically generates a decreasing relationship between prices and capacity. The second component $\mathcal{G}(M)$ captures the value of the recapitalization option created by limited liability and external financing. It represents the reduction in effective downside exposure associated with jump losses.

Proposition 6. *Within the projection-active region $\mathcal{P}$, the recapitalization-option component satisfies*

$$\mathcal{G}(M) = \frac{\lambda\phi}{l + \lambda\phi} \left[\frac{u(\underline{M})}{u(M)} - 1 \right].$$

Moreover,

$$\mathcal{G}'(M) = -\frac{\lambda\phi}{l + \lambda\phi} \frac{u(\underline{M})u'(M)}{u(M)^2} > 0.$$

Therefore, if the scarcity component satisfies $\mathcal{S}'(M) < 0$, a local price reversal occurs whenever $\mathcal{G}'(M) > -\mathcal{S}'(M)$.

The proposition formalizes the two opposing forces shaping equilibrium pricing. The scarcity effect tends to increase prices when insurer capacity is low, because capital becomes more valuable. In contrast, the recapitalization option creates a risk-taking incentive: when jump losses are frequent and insurers anticipate external support after severe losses, they may expand underwriting aggressively in financially constrained states.

C Projection-Switching in Robust Equilibrium

This appendix provides additional numerical evidence on how model uncertainty affects the transmission of projection switching into equilibrium pricing. The objective is to verify whether ambiguity eliminates or attenuates the pricing impact of the projection mechanism. We focus on the frequent moderate jump-risk environment $(\lambda, \phi) = (2, 0.2)$, which generates the strongest non-monotonic pricing pattern in the benchmark equilibrium. We compare the benchmark equilibrium with the robust equilibrium under $\theta = 20$.

The numerical results show that model uncertainty does not eliminate the underlying projection-switching mechanism. The benchmark equilibrium has a switching point of $M_k^b = 0.176$, while the robust equilibrium has $M_k^r = 0.257$. The difference reflects the endogenous adjustment of equilibrium prices and capacity allocation under ambiguity. Nevertheless, the existence of switching points in both equilibria confirms that the recapitalization mechanism remains.

To quantify the impact of ambiguity on the post-jump continuation-value transition, define the derivative jump at the projection-switching point as

$$\Delta_u \triangleq \left. \frac{d}{dM} u(\Pi(M^J(M))) \right|_{M_k^+} - \left. \frac{d}{dM} u(\Pi(M^J(M))) \right|_{M_k^-}.$$

The numerical results are $\Delta_u^b = -0.649$ for the benchmark equilibrium and $\Delta_u^r = -0.096$ for the robust equilibrium. Thus, ambiguity reduces the magnitude of the derivative jump in the post-jump continuation value by approximately 85%.

We further examine the corresponding change in the equilibrium price slope, defined as

$$\Delta_p \triangleq \left. \frac{dp(M)}{dM} \right|_{M_k^+} - \left. \frac{dp(M)}{dM} \right|_{M_k^-}.$$

The benchmark equilibrium generates $\Delta_p^b = -0.175$, whereas the robust equilibrium generates $\Delta_p^r = -0.008$. Hence, ambiguity reduces the magnitude of the price-slope jump by approximately 95%. Although the robust price function remains technically non-smooth, the associated kink is economically small and difficult to distinguish visually. The numerical evidence supports the interpretation that ambiguity attenuates the pricing distortion through endogenous adjustments in the robust equilibrium system rather than eliminating the underlying recapitalization mechanism.

D Calibration of Jump Risk Parameters

This appendix describes the construction of the market-disciplined calibration for the jump-risk parameters λ and ϕ . The objective is not to estimate the structural parameters of the equilibrium model, but rather to provide an empirical discipline for the order of magnitude of large-loss shocks considered in the model.

We use the annual global natural disaster damage data from the EM-DAT/Our World in Data database over the period 2000–2024. The dataset reports aggregate economic losses from natural disasters at the global level, which provides a broad measure of large-loss episodes that may affect insurers’ balance sheets. Since the data measure total economic damages rather than insured losses, we interpret them as a proxy for the frequency and severity of industry-relevant loss shocks. This reduced-form approach is consistent with the objective of calibrating the jump component, rather than estimating a detailed catastrophe loss model.

Let L_t denote the annual loss measure and define the normal annual loss scale as the sample median $L_{\text{normal}} = \text{median}(L_t)$. Using the 2000–2024 sample, the normal loss scale equals 0.1895% of world GDP. We identify severe-loss years as those in which annual losses exceed twice the normal loss scale $L_t \geq 2L_{\text{normal}}$. This threshold identifies three severe-loss years during the sample period: 2005, 2011, and 2017. The implied arrival intensity of large-loss shocks is therefore calculated as the fraction of severe-loss years:

$$\hat{\lambda} = \frac{N_{\text{extreme}}}{T} = \frac{3}{25} = 0.12.$$

This corresponds to approximately one major aggregate loss shock every eight years.

To calibrate the jump severity parameter, recall that ϕ in the loss process represents the incremental loss associated with a jump event, rather than the total loss in a jump year. For the three identified severe-loss years, the loss ratios are 2.381, 2.636, and 2.110. The mean severity ratio is therefore

$$\frac{1}{3} \sum_{i=1}^3 \frac{L_i}{L_{\text{normal}}} = 2.376,$$

implying the jump-size calibration $\hat{\phi} = 2.376 - 1 = 1.376$. Using the median severity ratio instead yields a similar estimate, $\hat{\phi} = 1.381$, indicating that the calibration is not driven by a single extreme observation. The following table reports the detailed annual global natural disaster loss series.

Table 3: Global Natural Disaster Losses and Extreme-Year Identification, 2000–2024

Year	Loss/GDP (%)	Relative Loss Scale	Extreme
2000	0.139	0.732	No
2001	0.086	0.454	No
2002	0.149	0.785	No
2003	0.178	0.937	No
2004	0.304	1.603	No
2005	0.451	2.381	Yes
2006	0.067	0.351	No
2007	0.126	0.667	No
2008	0.297	1.570	No
2009	0.073	0.385	No
2010	0.190	1.000	No
2011	0.500	2.636	Yes
2012	0.208	1.096	No
2013	0.153	0.807	No
2014	0.120	0.634	No
2015	0.103	0.545	No
2016	0.207	1.095	No
2017	0.400	2.110	Yes
2018	0.154	0.812	No
2019	0.113	0.597	No
2020	0.207	1.091	No
2021	0.261	1.375	No
2022	0.221	1.166	No
2023	0.194	1.022	No
2024	0.250	1.319	No

Notes: The loss series is obtained from the EM-DAT/Our World in Data global natural-disaster damage dataset. Losses represent aggregate economic damages rather than insured losses. Relative loss scale is normalized by the median annual loss over 2000–2024. An extreme year is defined as a year with losses at least twice the normal loss scale.

E Expected Underwriting Cycle Duration

This appendix describes the computation of the expected duration of underwriting cycles. Under the optimally chosen underwriting and pricing strategies, the aggregate capacity of the insurance

sector evolves according to the following dynamics:

$$\begin{aligned}
dM_{b,t} &= D(p_b(M_{b,t})) \left[\lambda\phi + (l + \lambda\phi)p_b(M_{b,t}) \right] dt - D(p_b(M_{b,t})) \eta dB_t - D(p_b(M_{b,t})) \phi dN_t \\
&= \underbrace{D(p_b(M_{b,t})) \left[-D(p_b(M_{b,t})) \eta^2 \frac{u'_b(M_{b,t})}{u_b(M_{b,t})} + \lambda\phi \frac{u_b(\Pi(M_{b,t} - D(p_b(M_{b,t}))\phi))}{u_b(M_{b,t})} \right]}_{\Psi_b(M)} dt \\
&\quad \underbrace{-D(p_b(M_{b,t})) \eta dB_t}_{\Sigma_b(M)} \underbrace{-D(p_b(M_{b,t})) \phi dN_t}_{\Omega_b(M)}, \quad M_{b,t} \in [0, \overline{M}_b],
\end{aligned} \tag{E.1}$$

$$\begin{aligned}
dM_{r,t} &= D(p_r(M_{r,t})) \left[\lambda\phi + (l + \lambda\phi)p_r(M_{r,t}) + \eta h(M_{r,t}) \right] dt - D(p_r(M_{r,t})) \eta dB_t^h - D(p_r(M_{r,t})) \phi dN_t^g \\
&= \underbrace{D(p_r(M_{r,t})) \left[-D(p_r(M_{r,t})) \eta^2 \frac{u'_r(M_{r,t})}{u_r(M_{r,t})} + \lambda^g \phi \frac{u_r(\Pi(M_{r,t} - D(p_r(M_{r,t}))\phi))}{u_r(M_{r,t})} \right]}_{\Psi_r(M)} dt \\
&\quad \underbrace{-D(p_r(M_{r,t})) \eta dB_t^h}_{\Sigma_r(M)} \underbrace{-D(p_r(M_{r,t})) \phi dN_t^g}_{\Omega_r(M)}, \quad M_{r,t} \in [\underline{M}_r, \overline{M}_r],
\end{aligned} \tag{E.2}$$

where the first process describes the dynamics of aggregate capacity under the physical probability measure $\mathbb{P}$, while the second corresponds to the dynamics under the worst-case measure $\mathbb{Q}^{h,g}$. Under the equilibrium conditions, the drift, volatility, and jump terms (denoted respectively by $\Psi(\cdot)$, $\Sigma(\cdot)$, and $\Omega(\cdot)$) can all be expressed in analogous functional forms.

Proposition 7. *Let $o \in \{b, r\}$ index the benchmark and robust equilibria, respectively. Define the infinitesimal generator $\mathcal{L}_o$ acting on a sufficiently smooth function $f(\cdot)$ as:*

$$\mathcal{L}_o f(M) = \Psi_o(M) f'(M) + \frac{1}{2} \Sigma_o(M)^2 f''(M) + \lambda_o \left[f(\Pi(M + \Omega_o(M))) - f(M) \right].$$

Denote by $T_{s,o}(x)$ the expected time for $M_{o,t} = x$ to reach the upper boundary $\overline{M}_o$. It satisfies:

$$-1 = \mathcal{L}_o T_{s,o}(x), \quad T_{s,o}(\overline{M}_o) = 0, \quad T'_{s,o}(\underline{M}_o) = 0.$$

Similarly, let $T_{h,o}(x)$ denote the expected time for $M_{o,t} = x$ to return to $\underline{M}_o$, which solves:

$$-1 = \mathcal{L}_o T_{h,o}(x), \quad T_{h,o}(\underline{M}_o) = 0, \quad T'_{h,o}(\overline{M}_o) = 0.$$

The expected duration of the soft market phase is $T_{s,o}(\underline{M}_o)$, while that of the hard market phase is $T_{h,o}(\overline{M}_o)$. The total expected duration of an underwriting cycle is therefore $T_{c,o} \triangleq T_{s,o}(\underline{M}) + T_{h,o}(\overline{M})$.

Figure E.1 presents the numerical solutions of the expected phase duration functions. The two curves correspond to the expected time required to reach the upper and lower capacity boundaries, respectively. These solutions are obtained from the boundary value problems defined above and are used to compute the expected soft-market, hard-market, and total underwriting cycle durations reported in the main text.

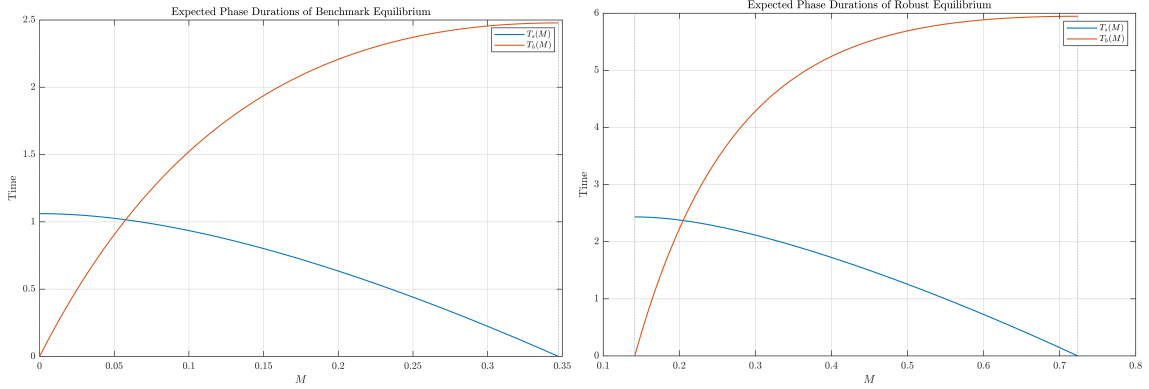


Figure E.1: Numerical Solutions of Expected Phase Duration Functions.

F Additional Model Extensions

F.1 Stochastic Jump and Compound Poisson Extensions

The baseline model assumes a fixed jump size ϕ for tractability and to isolate the economic role of discontinuous loss shocks. In reality, however, catastrophe losses are heterogeneous in severity, and different events may generate substantially different balance-sheet impacts. A natural extension is to replace the fixed-size jump process with a compound Poisson process:

$$dL_t = l dt + \eta dB_t + dJ_t,$$

where $J_t = \sum_{i=1}^{N_t} Y_i$ and Y_i represents the random severity of the i -th jump event.

Under this extension, the jump continuation term in the equilibrium condition would become $\lambda E_Y [u(\Pi(M - D(p)Y)) - u(M)]$, instead of the fixed-size expression $u(\Pi(M - D(p)\phi)) - u(M)$. The

balance-sheet mechanism identified in this paper would naturally extend to this setting : whenever the severity distribution has positive probability of sufficiently large losses, jump events can deplete internal capacity and trigger recapitalization through the projection operator. However, allowing stochastic jump severity would introduce additional heterogeneity into the loss process and may quantitatively affect the shape of the pricing function and the magnitude of nonlinear effects. A full analysis of compound Poisson extensions would therefore require solving the corresponding integro-HJB/HJBI system, which is beyond the scope of the current paper.

F.2 State-Dependent Robustness Concerns and Financing Conditions

The baseline model assumes that robustness concerns and financing frictions are constant over time. This assumption provides tractability and allows us to focus on how catastrophe-related jump losses affect equilibrium pricing and capacity dynamics. In practice, however, insurers' concerns about model uncertainty and the cost of external financing may vary with market conditions. For example, after large aggregate losses, insurers may become more concerned about future loss distributions, while the cost of external capital may increase when many insurers simultaneously seek recapitalization.

A natural extension is to allow the robustness parameter or financing wedge to be state-dependent, such as $\theta = \theta(M)$ or $\gamma = \gamma(M)$. Such an extension would capture how insurers' risk assessments and financing conditions change over the insurance cycle. However, it would require a more general equilibrium framework with additional state dependence and is beyond the scope of the current paper. The sensitivity analysis in Section 5.2 shows that robustness concerns and financing frictions are quantitatively important determinants of equilibrium prices and underwriting cycle durations. Understanding their endogenous variation over the insurance cycle remains an important direction for future research.

F.3 Heterogeneous Insurers

A further simplifying assumption is that insurers are homogeneous. Specifically, all insurers share the same information about loss dynamics and evaluate model uncertainty with the same degree of robustness concerns. In practice, however, insurers differ in their information sets, modeling capabilities, capital positions, and ability to absorb uncertainty.

A natural extension is to introduce a finite number of heterogeneous insurers indexed by $i = 1, \dots, N$, where insurers differ in preferences, robustness concerns, capital constraints, or beliefs about future losses. Related approaches have been adopted in the insurance literature, such as [Feng et al. \(2023\)](#), who characterize heterogeneous insurers under CARA preferences. However, incorporating such heterogeneity into a capacity-driven equilibrium framework following [Henriet et al. \(2016\)](#) and [Luciano and Rochet \(2022\)](#) is challenging, as individual underwriting and pricing decisions jointly determine aggregate market capacity. Developing a framework that aggregates heterogeneous balance-sheet conditions, risk assessments, and robustness concerns is therefore an important direction for future research.